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Children becoming expert symbol users

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Introduction

This chapter explores semiotics as a significant aspect of teachers' subject knowledge and reveals its central role in children's understanding of mathematical notations. It argues that this meaning making perspective can more effectively support children's early and developing use of the abstract symbolic language of mathematics (DCSF, 2008), deepening their understanding of calculations, problem solving and wider aspects of the mathematics curriculum as they move through school.

Chapter aims

Since both writing and 'written' mathematics are abstract symbolic languages, learning them can be complex for young learners. An important part of the 'Mathematics Specialist Teachers' programme' (MaST) in England is for teachers to understand *'how children learn mathematics, progression in mathematical understanding and recording and mathematical language*' (Edge Hill University, 2012). By the end of this chapter you should have an appreciation of the role of abstract symbols, appreciating children's active role in developing symbolic flexibility in mathematics.

- Your reading will be supported with findings from research helping you recognise how, through meaningful social contexts; children's informal prior knowledge can be extended and developed.

- Key features of successful learning highlighted within the chapter will help you appreciate how they contribute to learning as children move through the Primary school.

- The chapter will enable you to develop your understanding of some aspects of effective classroom cultures that enhance children's understanding of this significant aspect of mathematics.

Broad links to curriculum requirements

Early Years Foundation Stage

The current *Early Years Foundation Stage* (EYFS) informs practitioners that in

'Problem Solving, Reasoning and Numeracy', 'effective implementation' includes valuing, 'children's own graphic and practical explorations of (DfES, 2007: 61). It emphasises that children should:

- Create and experiment with symbols and marks
- … show their understanding of number labels such as 1, 2, 3
- Begin to represent number
- Use own methods to work through a problem' (DCSF, 2007: 64-8).

The *Williams Maths Review* (DCSF, 2008) made specific recommendations regarding children's understanding of abstract mathematical symbolism, devoting a significant section to *children's mathematical graphics* (Carruthers and Worthington, 2006) in the chapter on Early Years (although unfortunately omitting to emphasise its relevance to Key Stage 1). One of the Review's main recommendations resulted in the publication *Children Thinking Mathematically,* (DCSF, 2009) that we were commissioned to write.

Curriculum information – Key Stages 1 & 2
In the current National Curriculum 'using and applying number' for Key stages 1 and 2 is divided into sections headed *problem solving*, *communicating* and *reasoning* that include: 'Develop flexible approaches to problem solving… Make decisions about which operations and problem-solving strategies to use… communicate in spoken, pictorial and written form… using informal language and recording[1], then mathematical language and symbols' and 'explain their methods and reasoning when solving problems involving number and data'.

For Key stage 2 pupils should **'**find different ways of approaching a problem in order to overcome any difficulties… organise work and refine ways of recording… use notation, diagrams and symbols correctly within a given problem… present and interpret solutions in the context of the problem', (DfE website: 2011). At the time of writing, both the EYFS and the National Curriculum are under revision and it is to be hoped that guidance for teachers on these aspects are given greater attention.

Children's development of mathematical knowledge and understanding
In 1977 Ginsberg first documented children's difficulties with 'written' mathematics: subsequently Hiebert argued that 'many of the children's observed difficulties can be described as a failure to link the understandings they already have with the symbols and rules they are expected to learn. Even though teachers illustrate the symbols and operations with pictures and objects, many children still have trouble establishing important links' (Hiebert, 1984: 501). Hughes's seminal research into young children's difficulties with maths (1986) showed how children's own informal and intuitive representations play a significant role in 'translating' between their own representations and later 'school' mathematics. Hughes suggested that

[1] It is important to emphasise that *recording* places the emphasis on mathematical symbols, notations as *products* and is a lower level of cognitive demand than *representing* mathematical thinking (as in children's own *mathematical graphics*).

greater emphasis should be placed on children's early understandings by '… introducing symbols in meaningful communicative situations… If this idea can be communicated effectively to young children, it may have a profound effect on their subsequent mathematical education' (1983: 172).

Many other researchers have explored the significance of symbol use in mathematics (e.g. Gifford, 1990; diSessa, 1991; Bialystok, 1992; Cobb et al. 2000; Carruthers and Worthington, 2005, 2006; Brizuela, 2004; Ernest, 2006) acknowledging the centrality of children's own representations and written methods for mathematics.

Impact on understanding and achievement

Children's experiences in the Foundation stage clearly impact on their confidence and achievement in mathematics. FSP point 8 **'**Uses developing mathematical ideas and methods to solve problems' was particularly highlighted in the *Williams Maths Review*, emphasizing 'relatively few children' attain this point (DCSF, 2008: 41). Subsequently details of children's learning at the end of the Foundation stage were published, showing the lowest levels of achievement for FSP.8 were writing and *calculations* (DfE, 2010).

Since both writing and 'written' calculations are abstract symbolic languages it seems likely that their difficulties with symbolic understanding may be at the heart of this problem: this can lead to children who 'may eventually resort to coping strategies that alienate themselves from an understanding of number' (Munn, 1998: 70). Their confusion often becomes evident from 5 – 7 years, the point at which teachers demonstrate specific ways to 'record' and when increasing formalization of mathematical notations is assumed helpful. Evidence from Ofsted reports since 2002 (for example, Ofsted, 2008) repeatedly highlight the lack of children's informal written methods in Reception classes and throughout the Primary school and a need to strengthen the relationship between mental and written mathematics: they also raise concerns about the low levels of attainment with calculations, and in the area of 'using and applying mathematics'.

Introduction to concepts, issues and theoretical underpinning

Understanding, using, and communicating mathematics

This chapter is underpinned by socio-cultural and social-semiotic theory (e.g. Vygotsky, 1978; Kress, 1997) and is founded on our belief that all children can become confident symbol users and problem solvers in mathematics.

. 'Semiotics' describes the relationship between signs, meanings and cultures: this is the point at which this chapter begins.

Mathematical symbols, written calculations and charts support thinking and enable children to explore and communicate ideas. Children's own symbolic representations mediate understanding: they are acknowledged as 'symbolic' or cultural 'tools' that can be used to resolve particular psychological problems (Vygotsky, 1978), and are integral to mathematics.

> ‘The invention, use and improvement of appropriate symbols’, van Oers emphasises ‘are the characteristic features of the mathematical enterprise . . . the efforts of the pupils to get a better grip on symbols in a meaningful way should be considered one of the core objectives of education, especially in the domain of mathematics’ (2000: 136).

Research evidence

Based on evidence from her research with young children, Munn reached two important conclusions - that for young children progress in understanding symbolic functions depends on both writing and on graphical representations of number that children explore in a range of contexts, and that ‘The important development is the *function* of the symbol – its role in the child’s mental activity - and not in its form’ (1998: 69).

Pape and Tchoshanov propose that mathematical representations need to be thought of as tools for cognitive activity ‘rather than products or the end result of a task’ (2001: 124). Terwel et al. (2009: 28-9) argue that firstly graphical representation ‘closes the gap between prior knowledge and the material [children] are involved with. Secondly, it provides opportunities for creative engagement and ownership of conceptually difficult material. Third, it enables students to exercise their ‘meta-representational knowledge’, which is expected to be of value in the creation of new representations.

The evidence from a substantial body of research shows that a significant and positive feature of children’s engagement in developing personal representations to communicate their conceptual understanding, is their active involvement with such representations that can ‘develop *a sense of knowledge ownership*, which makes students feel free to transform this knowledge as the situation requires’ (Terwel et al. 2009: 28–9, emphasis added). Hoyles emphasises that mathematical thinking depends on students being ‘able to interact with abstraction’, arguing that ‘*symbols* shape how mathematics is expressed and ‘known’. Like Terwel et al. Hoyles also emphasises ‘that students *take ownership* of mathematical processes’ (2010: slides 2 & 3, emphasis in the original).

These features help pupils ‘learn how structuring processes develop’, enhancing their ‘capacity to generate new solution processes and to transform a representation according to changes in the situation, facilitating the construction of solutions to relatively new and unfamiliar problems’. Children are then ‘in a better position to understand pictures, graphs, schemes, models, or similar intellectual tools and are more successful in solving new, complex mathematical problems on relations and proportions’ (Terwel et al. 2009: 26-27; 41–2). Evidence from research by Terwel et al. show that these features also result in positive benefits for children aged 10-11 years of age, findings that are significant for Primary schools in England.

Becoming successful symbol users in mathematics

The philosophy underpinning our approach is rooted in the belief that children have a significant role to play in their learning, both in the understandings they bring and in their active engagement with graphical representations and symbols. From this perspective children's potential is realised through social contexts and in learning cultures rich in graphicacy and meaning making. The adults' role is pivotal in their learning. Adults create a rich learning environment that includes interactive displays of texts and children's graphics; adults model graphical representations and symbols for authentic purposes arising from children's play and every day experiences. Above all adults show that they value children's thinking and their graphical marks and representations by taking a close interest in their meanings and ideas.

Rather than drawing, maps, writing and mathematical notations being isolated 'activities' separated from personal meaning, or 'products' embedded in teacher-planned tasks, graphicacy becomes an integral and significant aspect of children's and adults' daily experience. In graphically and meaning rich contexts, children *choose* to use symbolic representations to support and communicate their thinking. As the case studies in this chapter show, immersion in such contexts support powerful individual and shared understandings of the abstract symbolic language of mathematics, leading to flexible use of calculations and problem solving.

Children's understanding of symbols and their functions are central to this perspective and align well with FSP profile point 8 'uses developing mathematical ideas and methods to solve practical problems' and 'using and applying number' of the National Curriculum. In our research we were able to chart children's developing skills as symbol users from birth to 8 years through our taxonomy, from early marks to calculations (see Carruthers's and Worthington, 2011a). Children's invented algorithms and notations support later understanding (Thompson, 2008), and shown by research evidence from Terwel et al. of children of 10-11 years (2009).

Case studies

The case studies begin with an example from a three-year old that may at first not seem 'mathematical': what may at first appear as casual or insignificant – at first intuitive, personal and informal marks and representations - have an important role to play in learning the abstract symbolic language of mathematics (Worthington, 2012).

Case study 1: introduction

This case study is from a nursery in an inner city Children's Centre and is an example of freely chosen and spontaneous pretend play (role-play), which has the potential to provide rich contexts for semiotic communication. One of its strengths is its social nature, allowing for shared meaning making and collaborative dialogue that enables both symbolic languages (speech and mathematical) to complement and support each other (Worthington, 2011).

Case study 1: Shereen – 3 years, 5 months

In Shereen's family shopping for food, preparing and sharing meals at home, in a restaurant or from a 'take-away' are especially valued cultural activities. Shereen draws on this cultural knowledge in her play.

At this point in her pretend play of a 'café with friends, Shereen asks her teacher Emma what she wants to eat, 'I'm writing chocolate bar, what you want?' Drawing some wavy lines (as writing) on her notepad, Shereen adds 'I've got rice, chocolate, chicken?' After taking Emma's order for rice she refers to its price, 'There you go: it's two, one, two'. Later Shereen returns again to ask Emma if she wants anything else to eat and when Emma replies that she doesn't want chicken, Shereen makes some marks at the top of the paper explaining 'Chicken' and then draws a cross nearby saying, 'It says 'x' – no chicken!'

Later again in their play Shereen returns once more to Emma, who says that she would after all like some chicken, but pointing to where she'd written a cross on her notepad, Shereen explains indignantly, 'Look! No chicken!' Then pointing to a tick she'd written by her drawing of a mushroom said 'Look! A tick, that mean we got some [mushroom]', adding You want ice cream? It's three, four. [2]

Figure 1: *Shereen's symbols*

Commentary

[2] With sincere thanks to all the staff of Redcliffe Children's Centre and maintained nursery, Bristol, for this case study and many other rich examples and insights.

Shereen drew on her ‘home’ knowledge of taking orders in a restaurant: she was clear that each item of food had its own price and used a range of graphical representations to represent meaning. Using drawings and writing-like marks to represent these items, Shereen also explored functions of abstract symbols using a ‘cross’ (to signify ‘No chicken!’) and a ‘tick’ (to signify ‘We got some’).

Symbol use

In symbolically rich learning cultures such as this nursery’s, young children often show remarkable understanding of abstract symbol use. Shereen’s symbolic communication was spontaneous and it is important to emphasise that her symbols had not been directly taught, neither had an adult intervened to make suggestions about how she might represent her orders. Vygotsky emphasised that ‘… teaching should be organised in such a way that reading and writing are necessary for something… must be something the child needs’. Moreover, writing ‘should be meaningful for children, that an intrinsic need should be aroused in them and that writing should be necessary and relevant for life’ (1978: 117-18): we believe that this is equally true for mathematical representations; it is clearly so for Shereen.

Communication is a significant aspect of semiotics, and Munn argues that ‘the analogy with emergent writing, where children bring their own meanings to communicative mark-making, seems convincing… The depth of the analogy depends on the children’s understanding of their numeric mark-making as *communicative*… If emergent number and emergent writing were truly similar, then this understanding would be present, because an understanding that symbols communicate meaning is a central element in progression’ (1997: 91, emphasis added).

At home Shereen is aware of the use of symbols to communicate, and staff in Shereen’s nursery are highly responsive to children’s interests and exceptionally good at ensuring that the learning environment indoors and out is rich in both mathematical symbols and texts. Number lines of different lengths are displayed and, as discussions about larger numbers arise, adults make the most of such opportunities to discuss number patterns (see figure 2 for an example from this nursery). Staff members also ensure that there are rich displays of children’s own graphicacy that include both original children’s examples and photographs of contexts of learning such as Shereen’s.

Figure 2: *Increasingly large numbers displayed in the nursery*

Many of the teachers and practitioners in Shereen's nursery have developed expert knowledge of graphicacy and symbol use that translates into their practice - and modeling symbolic representations is an important part of this. Based on her research, Munn argued that 'the association between numeral use and mental activity that incorporated symbols - suggest that powerful processes are at work whereby children form cognitive models… around the notations symbols that they have learnt from expert number users (1998: 66).

Reflection points

- Is the learning environment rich in meaningful mathematical print and texts?
- To what extent does display of children's symbolic representations support their understanding in your school?
- Discuss how these aspects might be developed.

Possible further action: modelling mathematics in context in Nursery and in Reception classes

- Discuss informal, daily opportunities for the teacher to model mathematics on a whiteboard through for example, quick drawings, numerals, informal or formal symbols or informal methods for calculating. These *short* spells of modelling can be highly effective should directly relate to the children's interests and experiences during play or everyday experiences and are best introduced spontaneously as opportunities arise.

Note: Significant changes in the children's representations and the strategies they choose to use will not be immediate and can only develop over time.

Case study 2

Case study 2 shows some strategies and personal methods children used to solve a personally meaningful problem in a Reception / Year 1 class.

Case study 2a: Frances – Year 1

The children had travelled by train to visit a pannier market in anticipation of planning a 'harvest market' at school. The following day they talked excitedly and Aaron commented 'I bet there's a million seats on that train!' He wondered how they could find out the total number of seats on the train and his classmates offered various ideas, one suggesting we 'ask the train people'. Empowered by this suggestion (and with adult help) Aaron phoned the local railway station, afterwards announcing that there were '75 seats in each carriage and 7 carriages on the train'.

The children who had chosen to help Aaron solve his problem used a range of informal methods. Frances explored several strategies to help her work this out. She then decided to represent 75 seats in a carriage: checking by re-counting she found that she had drawn 76 seats and crossed one out (figure 3).

Frances understood that this was a representation of only one carriage, and when invited to consider how she could help Aaron find the *total* number of seats on the train, her creative solution was to use the school photocopier to make an additional 6 copies of her drawing. When laid out end-to-end her representations of multiples of 75 on seven sheets of paper showed the sum total of '75 x 7' seats. Frances's combined strategies provided a valuable and powerful visual representation of repeated addition that children discussed for many weeks.[3]

[3] Case studies 2 and 3 are featured in Carruthers and Worthington, 2006.

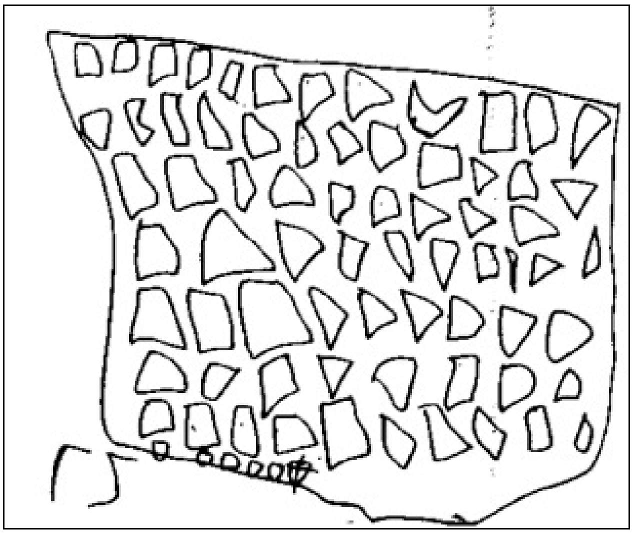

Figure 3: *Frances's train carriage*

Commentary

This example highlights children's decision-making in solving a problem using their own informal methods. Reflecting on the strategies that the children in this class used, Pound and Lee observe, 'these young children demonstrate many of the strategies identified by Polya [1957], strategies that 'successful problem solvers used' (Boaler, 2009: 179). They worked to understand the problem and they made plans. Some made charts of numbers, some drew their plans,' (Pound and Lee, 2010: 35-6).

Reflection points:

- To what extent are children's interests and ideas used as a starting point for an investigation or problem?
- Do children use a range of ways of representing and methods by choice? What do you believe might limit or encourage diversity and symbolic thinking?

Possible further action: for a Reception or Year 1 class

- In a forthcoming lesson provide blank paper for one group of children to use to support their thinking as they explore their own methods to solve a meaningful mathematical problem. It is important to *avoid demonstrating or providing any examples* of how to work out their problem.

- If the children have not been used to working in this way if may be helpful to explain that you are *especially interested* in *the children's own thinking and methods.* Ask if they have any thoughts about how they might begin? Does anyone else have a *different* idea?

- Before the lesson's end discuss the different strategies each used, listening to their various explanations and emphasising only the positive points.

- After the lesson assess the children's various responses, again only evaluating positive points. The second part of Carruthers and Worthington's taxonomy, 'calculations, children's own methods' may be of help to you (2011a: 76)

N**ote**: if the children have been dependent on being shown what to do they are likely to be hesitant the first time you try this.

Focusing on one group enables you to focus you attention on the children's thinking, strategies and representations.

Case study 3

In case study 3 a child adapts an informal taught method to solve a problem.

Case study 3: Miles – 7 years 5 months

Miles' s teacher posed a problem relating to an authentic situation – a picnic the class would have on their journey to their residential trip the following day. She showed the children a pack containing three nectarines, inviting them to work out how many packs she would need to buy for everyone in the class (a total of 26).

Miles decided to use an empty number line, and taking a piece of A4 paper he drew a horizontal line across the sheet. Beginning on the right of his paper Miles started with jumps of '3' but soon realised that by using his paper in this position he would have insufficient space to complete his calculation. Miles adapted his jumps, twice doubling his jumps of 3 to arrive at his answer of '9 packs' and acknowledging the surplus nectarine in his answer.

Insert Figure 4 near here

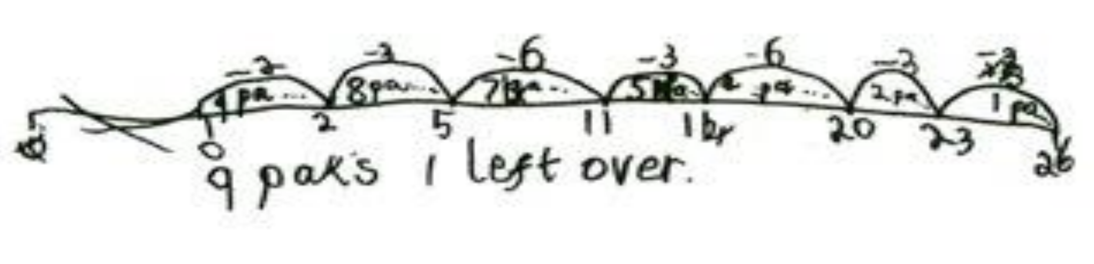

Figure 4: *Miles - calculating the number of packs of nectarines needed*

Commentary

Miles's response showed his flexible thinking, enabling him to adapt this visual method for his purpose of division as repeated subtraction. His peers used a range of different strategies including writing multiples of 3 up to 27 and mentally subtracting 1; drawing repeated groups of three 'stick' people in 9 boxes (up to 27), and then crossing one person out. Ann also chose to use an empty numberline, using regular jumps of 3 up to 27, and then a jump back of one to reach '26' explaining, '9 packs 1 left over'.

Originating in the Netherlands, the 'empty number line' was introduced by the National Numeracy Strategy (DfEE, 1999) and continues to provide children with a flexible tool for calculating. Gravemeijer et al. acknowledge children's own methods as significant in supporting 'an emergence of meaningful symbolizations that arise during collective negotiations' (2000: 241). The culture in the classrooms in case studies 2 and 3 reveals other strategies identified by Polya, such as children asking themselves 'what is involved in this problem?' Polya also taught teachers to encourage children to restate the problem in their own words, to use a drawing or diagram that might help them understand the problem, (1957).

Reflection points

- Reflect on the role of collaborative discussion in supporting children's developing representation and strategies in maths, and how you might develop this with individuals and with groups.

Possible further action: for a Key Stage 1 class

1. Refer to 'possible further action (see p. 11). Since these children are older and more experienced it is likely that some of them will choose to use or adapt a 'taught method' such as the empty numberline to help them with their **calculations** or **to solve a problem.**

2. **Data handling** offers rich alternative contexts, enabling children to develop their understanding of a range of ways to collect, represent and analyse data in open ways.

- When they have collected their data, invite the children to discuss what they've done and to represent their findings – again in their own ways (e.g. using drawings, numerals, charts, words, etc.,) – but without the teacher making any specific suggestions.

- For either group activity end the session by sharing some of the group's examples with the whole class. Invite other children (*not* those in your small group) to give positive feedback as they consider the strategies used, or read and analyse the data represented.

Author commentary
The case studies are intended to show how the approach outlined in this chapter enables children build on their experiences of symbol use and mathematical notations, and confidently use flexible methods to communicate and calculate. Children's symbolic understanding and methods mature and become increasingly efficient through Key Stage 1 and provide rich evidence for assessment. As young children's informal, personal notations move 'from unstructured mathematical knowledge towards 'mathematical understanding that is [increasingly] structured and organised using familiar and strongly formed tools' (Poland, 2007: 26).

Summary

The ways in which we teach mathematics are significantly shaped by our views: for example, a widely held view is that mathematics is a 'difficult' subject (especially for young learners), and due to its abstract nature written notation is 'hard' to learn. With good intentions this is often interpreted for younger children as simple tasks that are often devoid of meaningful contexts and involve only small numbers and quantities. Teachers plan activities such as copying or tracing numerals, colouring in, or depend on teacher-given models for calculations. In Key stage 1 a common practice is to demonstrate and explain, and for children to practice. Such activities and tasks result in Reception and Key stage 1 privileging 'correctly' written numerals, horizontal calculations, correct use and positioning of signs such as '+' and '=' and neatness *above mathematical thinking and understanding.* The outcomes of such experiences are particularly evident as children move through Key stage 1 since they have little *sense of ownership* referred to earlier.

The features highlighted here confirm their potential in supporting children's success in **'**Uses developing mathematical ideas and methods to solve problems' in the Foundation Stage, and 'using and applying number' in the National Curriculum. It is expected that by now your reading and reflection, combined with some 'possible points for action' and key features of successful learning discussed within the chapter, have helped you to appreciate how they contribute to the continuum as children move through the Primary school and lead to deepening understandings of the abstract symbolic language of mathematics.

This chapter should enable you to develop your understanding of aspects of effective learning cultures and enhance children's understanding in your school. The perspective explored in here rests on a belief that all children can be curious, competent and confident learners of mathematics. The aspect of mathematics explored in this chapter is one in which teachers can make a very real difference, and where there is potential for children to become expert symbol users and powerful makers of mathematical meanings.

Links to further reading:

- Carruthers, E. and Worthington, M. (2006) *Children's Mathematics: Making Marks, Making Meaning*. London: Sage Publications. (2nd Ed).

This book provides a thorough introduction to the development and range of young children's mathematical representations in the birth – 8-year age group, illustrating how children make mental connections between their own early marks and mathematical symbols, and how these relate to children's own written methods for calculations.

- Carruthers, E. and Worthington, M. (2011a) *Understanding Children's Mathematical Graphics: Beginnings in Play.* Maidenhead: Open University Press.

Beginning by exploring the relationship between pretend play and children's graphical representations, this publication includes numerous examples from children under three years and up to six years of age. They are drawn from teachers and practitioners who are developing their understanding and pedagogy to support children's mathematical thinking and graphics.

- Carruthers, E. and Worthington, M. (2011b) *Understanding Children's Mathematical Graphics: Supporting early Mathematical Thinking.*

This professional development pack addresses many aspects explored in this chapter and is suitable for the Foundation stage and KS 1.

- Boaler, J. (2010) *The Elephant in the Classroom: Helping Children Learn to Love Maths*. London: Souvenir Press.

Boaler explores some of the issues and problems discussed earlier in this chapter, with particular reference to Primary schools in England. She emphasises that in her research, effective teachers 'valued all of the different ways of being mathematical and students were not simply repeating procedures.' Teachers encouraged pupils 'to communicate in different ways, to decide on the appropriateness of different methods and to adapt methods in order to solve problem' (2010: 76-7).

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