Children’s Mathematical Graphics: Overview

Maulfry Worthington

Introduction

Origin

We originated the term *children's mathematical graphics* to describe the range of children's own mathematical marks and representations that we found (see for example, Worthington & Carruthers, 2003; Carruthers and Worthington 2005 & 2006). We collected hundreds of examples within our teaching in the birth - 8 year age-range and from children in their homes. More recently examples come from teachers and practitioners who have begun to explore children’s mathematical graphics in their own settings and schools, and are gathered through current research in which we are engaged.

Young children use a range of visual representations that may include scribbles, drawings, writing, iconic marks and invented (personal) and standard symbols. They use their own mathematical representations to help them think about and communicate meaning and to explore specific symbols and calculations: (see for example *Gallery 1* on our website), data handling and other aspects of the mathematics curriculum.

Our work on children’s mathematical graphics has grown from our many years' experience as teachers of young children and research into their development. Our extensive research includes research with children, parents, teachers and practitioners - in homes, nursery and schools across the 0 - 8 year age range.

The written language of mathematics

Written mathematics is vital aspect of the curriculum as children move through school, yet evidence has shown how written mathematics causes the greatest challenge for children (e.g. Ginsberg, 1977; Hughes, 1986; Carruthers and Worthington, 2006); *see also link - in references, below*). Vygotsky (1978) referred to written symbols as ‘symbolic’ or ‘cultural tools’ and mathematics as a subject has been described as ‘really a matter of problem solving with symbolic tools’ (van Oers, 2001, p. 63). Children’s mathematical graphics is a socio-cultural approach, recognising the experiences and understanding all children bring, and the importance of the culture of their early childhood setting.

As they move from the Foundation Stage into school, it may *appear* that most children understand written calculations and other aspects of written mathematics, yet for many children this understanding is likely to be only superficial. What has been missing is a means of ‘bridging the gap’ that Hughes (1986) identified, so that children can make sense of the written language of mathematics by ‘building on what they already know and understand’ (EYFS, 2007). Our evidence is that children’s mathematical graphics can do just this. Teachers and practitioners focus on the strengths of individual learners supporting them as they move forward in their understanding.

‘Practical’ mathematics

Practical mathematics (e.g. with blocks, sand, water and games) provide valuable mathematical experiences, but are not *directly* related to children’s understanding of *written* mathematics. Children learn about written mathematics through using their own ‘written’ mathematics (their own marks and representations): it is very difficult for them to understand written mathematics at a deep level if they only copy what adults show them to do, or are asked to write their mathematics in a particular way.

Children’s mathematical graphics supports their development and helps them understand the abstract symbols of mathematics. It helps them develop their understanding of written methods of calculations, problem solving, data handling - and all areas of written and represented mathematics. Our research has shown how children’s own graphics bridge their mental and written methods.

The importance of play

Children’s ability to make meanings with marks, symbols and representations have their roots in imaginative play in a diverse range of contexts and with many different resources. This ability to make personal meanings in very early childhood was identified by Vygotsky and in recent years has been explored from a multi-modal perspective (e.g. Kress, 1997 and Pahl, 1999). The emphasis with children’s mathematical graphics is very much on children making sense of the written language of mathematics and effective pedagogy to support their thinking.

When the culture and ethos in Nursery settings and Reception classes supports rich play; when children’s own meanings are valued and adults tune children’s marks and representations children will spontaneously choose to use their own mathematical graphics for self-chosen purposes within their play. Later in Reception teachers will sometimes plan adult-led groups for mathematics in which they solve problems: providing blank paper will allow children to explore and extend their thinking about the mathematics. As they move into Key Stage 1 and there will be an increasing focus on adult-planned mathematics lessons, and children will increasingly combine their own representations with standard symbols as they move towards standard forms.

Representation – or recording: process – or product?

The emphasis in *children’s mathematical graphics* is on children’s own *mathematical thinking, meanings* and *understanding* about the mathematics and all aspects of written mathematics and symbols. Since children can choose to represent their thinking in their own ways, their mathematical graphics are totally inclusive.

The value of using paper to explore their thinking is that their *own representations* support understanding by allowing children to see their thinking reflected back to them. In a sense children's mathematical graphics are their mental methods – on paper. Using their own mathematical graphics helps young children to 'translate' between their early informal marks and the standard symbols and written language of mathematics. If they are to develop deep understanding, children need to be free to choose how they will represent their mathematical thinking that best fits their purpose, the particular mathematical context or calculation they are exploring, or the problem they wish to solve.

In using their own mathematical graphics the emphasis shifts from recording to *processes of mathematical thinking* (creative thinking, reasoning, meanings, understanding, problem solving, negotiation and co-construction of understanding) rather than *products* (recording something done practically). Real contexts for their mathematics will allow children to make greater sense of their mathematics when they explore their thinking through their own representations.

Recording what they did following a practical activity has limited value and involves lower levels of thinking. Children do not need to record mathematics if they can do it mentally; neither do they need to record something they have worked out in a practical context. *Recording* places the emphasis on marks and drawings as a *product* and is a lower level of cognitive demand (thinking) in mathematics. The difference between *representing mathematical thinking* and *recording* is one of quality and depth of thinking.

Pedagogy

Key aspects of pedagogy include:

- a learning environment rich in play opportunities, mathematical print and potential
- sensitive adult support informed by knowledge of the mathematics and the children's developing understanding
- adults who understanding children's development of early 'written' mathematics (from birth to 8 years)
- a range of ways of modeling *(see references, below)*
- Collaborative dialogue is central to the children's mathematical thinking, as they discuss their mathematical graphics and the mathematical content (see Carruthers and Worthington, 2008). This allows meanings children's thinking and meanings to be negotiated and their understanding about the mathematics and ways of representing and mathematical symbols become clearer.

Understanding, supporting and assessing

In a major piece of research we conducted, we analysed over 700 examples of children's mathematical graphics from children in the home, nursery settings and schools. This important and in-depth analysis uncovered the development of young children's own mathematical graphics and written calculations for the first time. It reveals the complexity and creativity of young children's mathematical mark-making and their astonishing ability to make meanings though their graphics – and how these support their mathematical thinking. This research led to the taxonomy that charts children's development from birth to eight years (see website for download of the taxonomy).

Children's mathematical graphics also have tremendous value for teachers since they reveal each child's thinking about all aspects of written mathematics. Annotated pieces also offer an invaluable tool for assessment in mathematics when used with the taxonomy.

Transition – continuity in 'written' mathematics

Children's mathematical graphics lay a firm foundation for the 'school' mathematics they will meet (in England, following the Early Years Foundation phase) as they move into Key Stage 1. Rather than mathematical experiences being unconnected, it is vital that problem solving and mathematical concepts are at the heart of their mathematics. Using their own graphics to support their mathematical thinking builds on what they already know and can do and helps their development towards the standard, abstract written language of mathematics. It is important that children continue to build on this early understanding as they move through Key Stage 1 to ensure that they move towards the standard written symbols, layout of calculations and forms of data handling with ever deepening understanding.

CPD

Supporting effective professional development is at the heart of our work. Teachers and practitioners can develop their understanding through the *Children's Mathematics* website and through reading publications (see: http://www.childrens-mathematics.net/publications.htm). In addition to conferences and occasional courses, we particularly encourage teachers and practitioners to develop their own *Children's Mathematics Network (CMN) groups* in their locality. These groups are 'owned' by those in the group and support rich, collaborative learning. CMN local groups support professional understanding of this important aspect of mathematics and are having significant impact on pedagogy and children's mathematical understanding.

Children's mathematical graphics: evidence-based research

Our work is based on extensive, evidence-based research with children, teachers and families and within the context of homes, nurseries and schools. We advocate a spirit of freedom and creativity for teachers and more importantly, the freedom for children to explore their own meanings in creative ways.

Our aim is to hear the voice of the child and to support effective pedagogy for this significant aspect of mathematics in this phase.

Notes:

1. We chose the term *children's mathematical graphics* as more appropriate (than 'emergent mathematics') to describe the full range of marks that young children use as they explore their mathematical thinking. Our reasons include:

- Writing and written mathematics are *not* the same - the subject content, the children's thinking about that content and the ways in which they are represented are inherently different. In emergent writing children are focusing on the letter-symbols and their relationship with sounds and words rather than numbers, calculations, ways of representing data, measurement and space and shape. In writing letters are gradually used to build words that together communicate meanings about content. In mathematics children use a wide range of marks, symbols and representations, firstly to help them think about the mathematics with which they are engaged and sometimes to communicate their mathematical thinking.
- Neither early writing nor early written mathematics emerges on their own: there is an active role for teachers and practitioners in children's developing understanding of both symbolic languages (writing and mathematics). Children need (and deserve) sensitive support from practitioners and teachers who understand their development of both languages and know how they can effectively support the children as they move towards standard forms.
- In the USA the term 'emergent mathematics' is used to refer to all aspects of mathematics in early childhood – rather than only to early 'written' mathematics
- Whilst we discuss children's *marks* and *mark-making*, this term is more commonly used with reference to early *writing*

Children's mathematical graphics include early mathematical marks; numerals; mathematical signs; tally-type marks; drawings; diverse ways of representing data and personal ways of representing shape, space and measurement - as they move towards standard forms.

2. We do *not* advocate young children sitting at tables engaged in 'written' mathematics every day. We support the full range of contexts and experiences for mathematics, beginning with play, and developed over time through dialogue and adult interaction. Towards the end of Reception and in KS1 adults may plan some adult-led groups in which children use paper to support their thinking.

3. Resources: children's mathematical graphics draw on children's imagination and creativity (their meaning-making) to support their mathematical thinking with symbols. In terms of resources it requires only blank paper and pens or pencils. <u>Please note:</u> *published schemes and adult-prepared materials do not support this approach.*

4. Written mathematics: official guidance: children's mathematical graphics provide a seamless means of supporting children's developing understanding of the written language of mathematics throughout the entire Foundation stage and Key Stage 1 (0 – 8 years). They fully support official guidance of the Early Years Foundation Stage (EYFS) for *Problem solving, reasoning and numeracy* and the Primary Strategy for mathematics. This way of working also supports ‘Point 8’ of the *Foundation Stage Profile* (2003); *Using and applying mathematics* of the Primary mathematics curriculum and official guidance on teaching written calculations from QCA *(see references, below).*

References

Carruthers, E. & Worthington, M. (2005) ‘Making sense of mathematical graphics: the development of understanding abstract symbolism’ *European Early Childhood Education Research Association Journal*, (EECERA) Vo 13, No.1 (pp. 57 – 79).

Carruthers, E. and Worthington, M. (2006) (2nd ed.) *Children’s Mathematics: Making Marks, Making Meaning*. London: Sage Publications.

Carruthers, E. and Worthington, M. (2008) 'Children's mathematical graphics: young children calculating for meaning' in I. Thompson, (Ed.) (2008) *Teaching and Learning Early Number*, Maidenhead: Open University Press, (2nd ed.): *forthcoming.*

Ginsberg (1977) *Children’s Arithmetic*. New York: Van Nostrand

Hughes, M. (1986) *Children and Number: Difficulties in Learning Mathematics*. Oxford: Blackwell.

Kress, G. (1997) *Before Writing: Rethinking the Paths to Literacy*, London: Routledge.

Pahl, K. (1999) *Transformations: Meaning Making in the Nursery*, Stoke-on-Trent: Trentham Books.

QCA (1999) *Teaching Written Calculations*; London: Qualifications and Curriculum Authority.

Van Oers, B. (2001). Educational forms of initiation in mathematical culture. *Educational Studies in Mathematics*, vol.46, 59-85.

Vygotsky, L.S. (1978). *Mind and Society: The Development of Higher Mental Processes*. Cambridge, MA: Harvard University Press.

Worthington, M. & Carruthers, E. (2003), *Children’s Mathematics, Making Marks, Making Meaning*. London: Paul Chapman Publishing.

- ‘*Effective Modelling’:* http://www.childrens-mathematics.net/pedagogy_modelling.pdf
- *‘Challenges young children experience with written mathematics’*: http://www.childrens-mathematics.net/pedagogy_challenge_children.pdf

---