Maulfry Worthington: published in Maths Coordinator's File, 25. *Mathematics Association:* May 2007.

Exceptional children: researching the young child's mathematics

> ***'The children in these studies are not exceptional, or rather, they are exceptional in every conceivable way'***,
>
> (Seymour Papert - *Mindstorms,* 1993, p. 13).

Introduction

This article describes some of the research that grew from our teaching throughout the whole of the Early Years age-range (3 – 8) and was eventually published in our book *Children's Mathematics* (Carruthers and Worthington, 2006). For a number of years we have been researching the early marks and drawings young children use to explore, represent and communicate their mathematical thinking: we term these representations *children's mathematical graphics.* Their own graphics reveal young children as powerful meaning-makers able to build deep understandings of the 'written' language of mathematics. Rather than a new approach to teaching, this work points to a new *vision* for young children learning mathematics.

Back to the future

At a conference I attended in 2000, Martin Hughes raised some questions about the newly introduced Numeracy Strategy and questioned the extent to which it would help young children better understand mathematics.

Ten years previously as classroom teachers we had been founder members of a local group of teachers exploring mathematics. We had been excited by what young children were achieving through an *emergent writing* approach and had begun to consider whether mathematics might be explored in a similar way. However we looked too wide and as a result our original question remained largely unanswered. Eventually other members of the group left to pursue different interests in their teaching and only Elizabeth and I remained.

On returning home from the conference in 2000, Martin Hughes's question hung in the air. Elizabeth and I arranged to meet to discuss whether we might begin some further research and focus on the children's own representations.

'Building on what children already know and can do'

In our classrooms we had encouraged children to use their own mathematical graphics for many years. This was in contrast with the usual approach at the time, to ask children to 'record' something they had already done in a practical context or to complete worksheets.

We wanted to understand what children intended their marks and symbols to *mean* mathematically and to see how this helped them as they began to calculate and explore other aspects of mathematics. We also hoped to identify the development of their marks.

The numerous pieces we had collected from nursery, Reception and Key Stage 1 included many that children had created spontaneously within their play and others in the context of adult-led groups or class lessons for every aspect of mathematics. To these we added recent pieces from other classes where we were invited to teach and some from younger children in our own families.

Researching early 'written' mathematics
Following the publication of Marie Clay's research into children's writing development in 1975, considerable interest grew that led to 'emergent' or developmental writing and influenced teachers' practice across the country. This research has continued during the past thirty years. An emergent writing approach recognises, supports and values young children's earliest marks as meaningful and valuable in their development as young writers.

In contrast there have been few studies of children's early 'written' mathematics. Hughes's research (1986) was an exception and explored some of the difficulties young children experience with the spoken and written language of mathematics. It identified the considerable gap that must be bridged if young children are to become confident with standard written mathematical language and calculations.

Older children and adults use the written language of mathematics without always realising how complex it can be for young children to learn a symbolic language such as mathematics: they may appear to understand but may do so in only a superficial way. The official advice for teachers is clear about the disadvantages of introducing and expecting standard symbols and calculations too early, emphasising 'it is easy to be misled by children who start to use standard forms of recording too early, into thinking that they necessarily understand what they have written (QCA, 1999b, p.19). Our research has shown that expecting such standard forms too early actually *prevent* children understanding them at a deep level. An emphasis on 'products' such as counting and colouring-in or completing a page of 'sums' is also sometimes seen as evidence of children's understanding, although in terms of assessment they are likely to tell us only who did not understand what to do or write.

Hughes's research showed how children could use their own marks to represent numerals and calculations in ways that they could understand. However in spite of subsequent encouragement for teachers to support a more flexible approach to the beginnings of 'written' mathematics (QCA, 1999 and 2000), Hughes's findings appeared to have seldom influenced practice in Reception and Key Stage 1.

Tracing the development of children's mathematical graphics
By 2001 we had amassed over 700 pieces of children's mathematical graphics.
At first we identified the range of ways in which children chose to represent their thinking, identifying the *forms* of children's mathematical graphics: these included three Hughes had found (pictographic, iconic and symbolic) in 1986.

Forms

- **Dynamic:** these are lively and energetic marks that suggest action: they are often spontaneous and have a fresh quality about them
- **Pictographic:** this form is like a picture, usually when the child is representing something she can see
- **Iconic:** children use one mark (or *icon*) to stand for one item that they are counting or calculating. Iconic marks or drawings are based on one-to-one counting. Young children use many different icons, rather than only tallies

- **Written:** children use emergent writing, letters, words or sentences to represent their mathematical meanings
- **Symbolic:** at this point, children use standard written numerals and mathematical signs by choice in their calculations and other aspects of the mathematics curriculum

Whilst these forms are not strictly hierarchical, we found that by five years of age many children are likely to choose to use some *symbolic* forms with understanding. As we looked closely at the different strategies that children used we became increasingly excited by the depth of their thinking that was revealed through their mathematical graphics.

There has been a great deal of research into young children's mathematical understanding (e.g. Gelman and Gallistel, 1978); Ginsberg (1989), Sophian, (1996); Nunes and Bryant, (1996). However there had been none on the development of children's informal written mathematics. Through a gradual process of sorting and comparison of hundreds of examples, we began to see that there were a number of different dimensions. We have described our methodology for analysing the children's mathematical graphics elsewhere (Carruthers and Worthington, 2005; 2006). Our analysis eventually led to the taxonomy we developed (Figure 1.), revealing children's mathematical graphics development from birth to eight years and providing new insights into young children's understanding of mathematics. Teachers are now finding the taxonomy invaluable for assessing children's mathematical graphics in their own settings and classes.

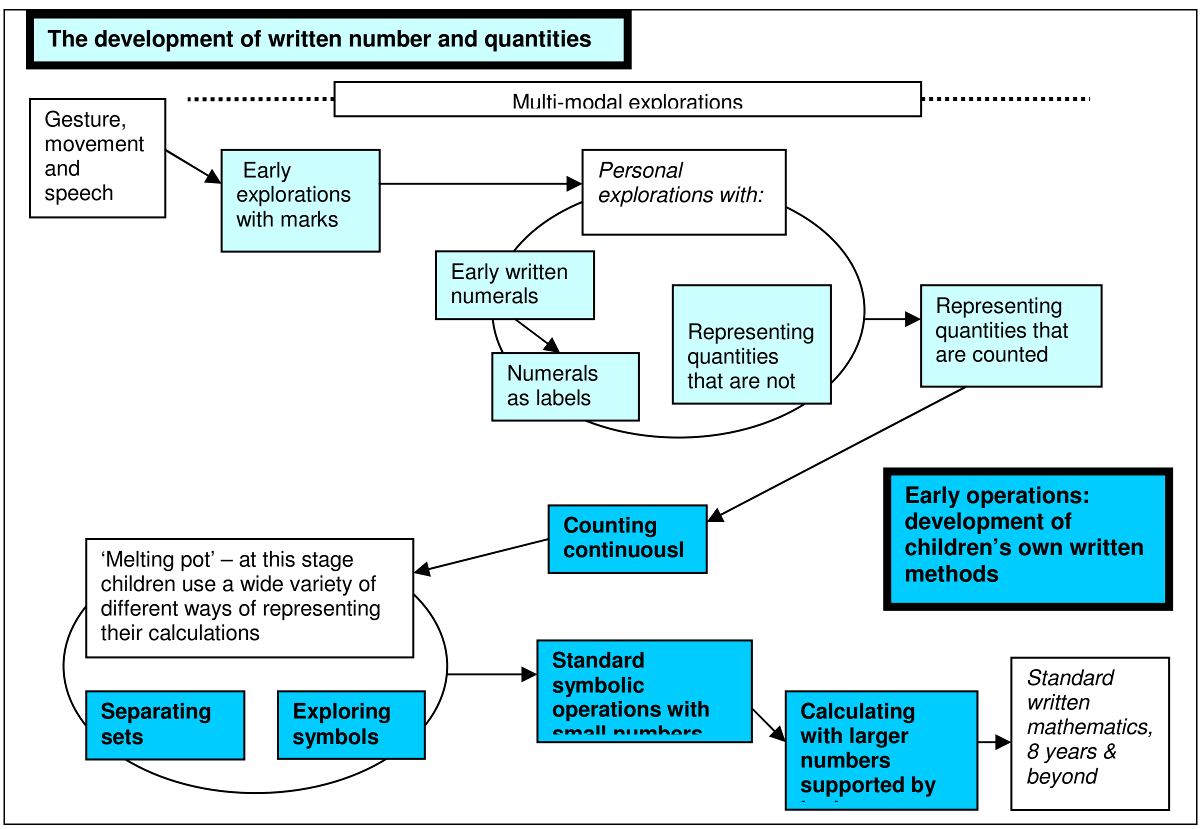
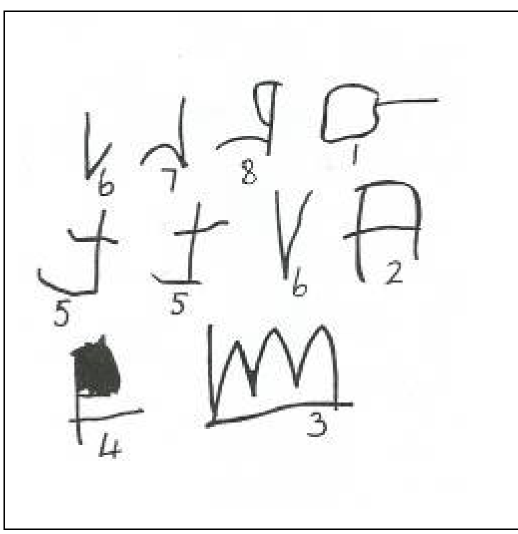


Table 1.

Development begins within children's earliest marks. Between the ages of three and four years of age children begin to identify separate meanings for their marks, naming some as written messages or as drawings: to others they attribute mathematical meanings. These marks develop for different purposes and children use them in many different contexts where they are thinking about numerals and quantities.

Table 1: The development of written number and quantities

Dimension	*Example*
Early explorations with marks: children's earliest marks arise through gesture and actions	*At just three years of age Matt loved to make marks and draw. One day he explained many pieces he had made in quick succession. Drawing two curving lines on one piece of paper he 'read' 'I spell 80354'*
Early written numerals: Children refer to their marks as numbers and explore ways of writing numerals.	*Alex (figure 2) was in his first term in the reception class. He was very interested in writing and in play spontaneously wrote a series of his own numerals that he then read to me. His symbols showed that he already knew a lot about standard written numerals: he had adapted some he had seen and generated his own rules about how to write them*
Numerals as labels: They use the numerals they know to 'label' things within their play, things they make or draw.	*Jay (3 years and 8 months) was drawing. Slowly she drew a curved shape and joined the ends. Pausing to admire what she had done she remarked in a surprised voice 'eight! My brother's eight'!*
Representing quantities that are not counted: They make many (uncounted) marks to represent a large quantity of something seen or imagined.	*In the nursery, Charlotte held a number of felt-tip pens in each hand, (figure 3) making many dotted marks, over the entire surface of the paper. She called out to her teacher 'Look! I'm doing hundred and pounds!' She was making connections with the quantity of dots she made and her thinking fits into her sense of a large quantity*
Representing quantities that are counted: Children count the marks or items they have represented, or represent items they have counted.	*In the nursery Jenna and her friend loved to play in the graphics area. One day Jenna chose lots of different coloured pens to make vertical columns of lines which she called 'raindrops'. She counted the fifteen individual marks as she moved her finger from one to another*



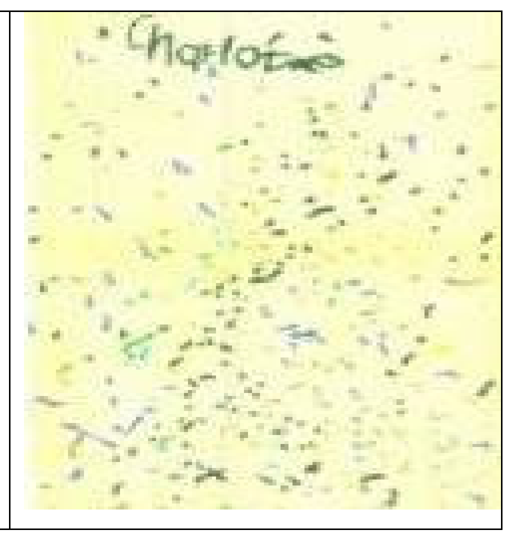


Figure 2. Alex's numbers **Figure 3: Charlotte's 'hundred and pounds'**

These aspects (in table 1) are the foundations of all the *standard* written mathematics that children will meet as they move through school. The range of representations and strategies that children use is huge. They move between the different dimensions, exploring aspects of mathematics that are of interest to them and meaningful within their play. Once children are *representing quantities that are counted*, they begin to explore calculations and further analysis revealed how their own written methods develop.

Table 2: Early operations: the development of children's own written methods

Dimension	*Example*
Counting continuously: children represent and count items, using a range of strategies for addition and subtraction	*Alison represented the seven children in her group and their seven teddies, who would be coming to the 'breakfast café', representing them as a continuous line of numerals*
Separating sets: children begin to represent the two sets in their calculations as separate	*Fred drew a line between the two sets of grapes that he was adding*
Exploring symbols: at this point children are search for understanding about symbols: they use their own invented or adapted symbols to help them understand the role of standard mathematical signs such as '+' and '='	(*See figure 4): this was the first time that Barney had explored subtraction as a 'written' calculation. He used several ways to help him think about the operation. His invented arc of arrows helped him understand the subtraction symbol as 'taking away'.*
Standard symbolic operations with small numbers: children increasingly integrate and use standard numerals and operators within their calculations	*Eleanor used standard symbols and signs in horizontal calculations to help her think about the operation of subtracting beans, Finally running out of beans she collected some counters and returning to an early method, writing '4 contus t 1 = 3' (4 counters take 1 equals 3')*
Calculating with larger numbers supported by jottings: the written methods they use to support their thinking often includes taught jottings	*To help her work with larger numbers, Alison drew on a range of informal strategies such as tallies and repeated addition and some she had been taught, including rounding up and subtraction (figure 5).*

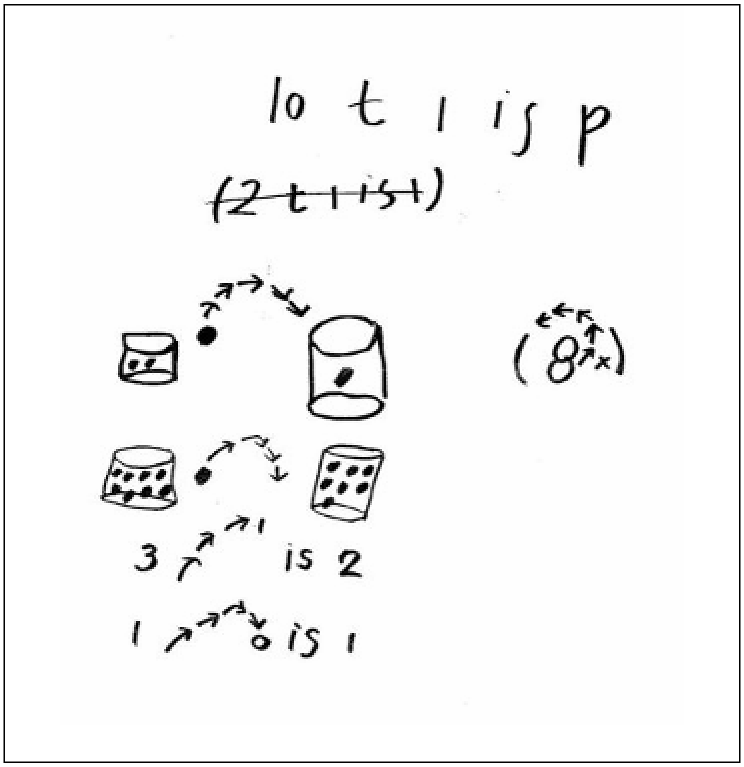


Figure 4: Subtracting beans

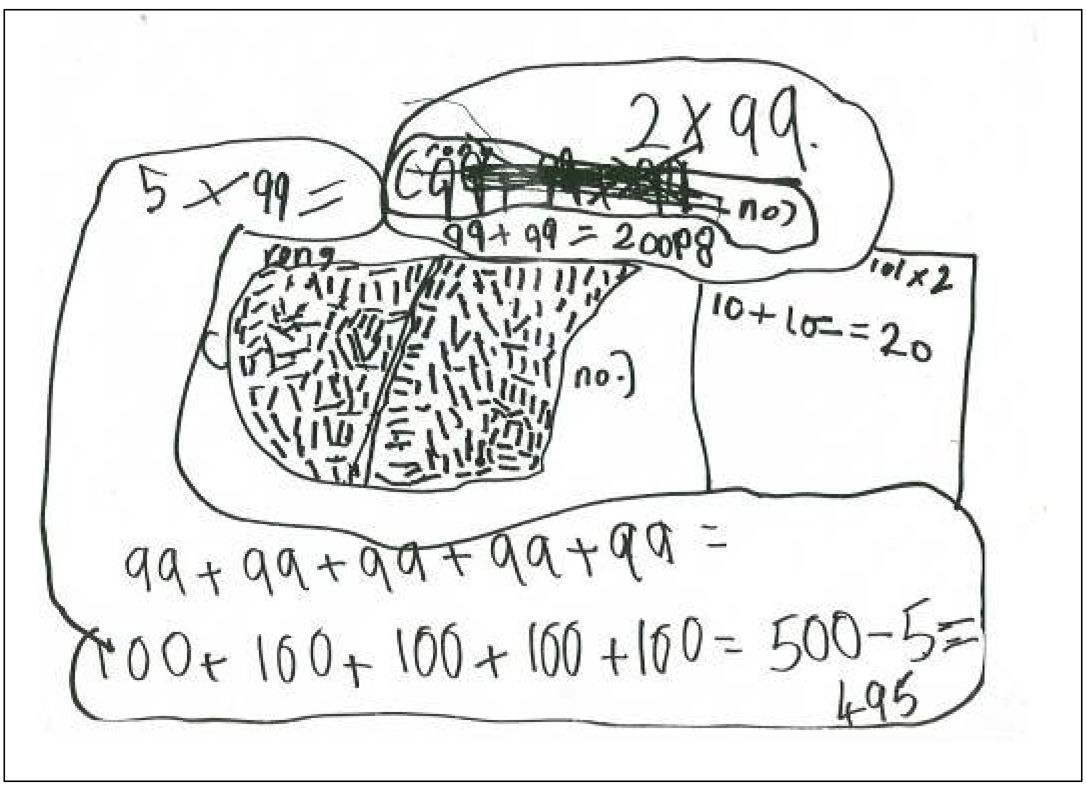


Figure 5: Alison's '99 times table'

Children show their amazing ability to solve problems, and their own representations help them think explore strategies to do this (see figures 5 and 6). Kamrin (figure 6) developed some highly original ‘tweedle birds’ and shared eight ‘eggs’ between them to help him work out if 8 could be divided equally between two.

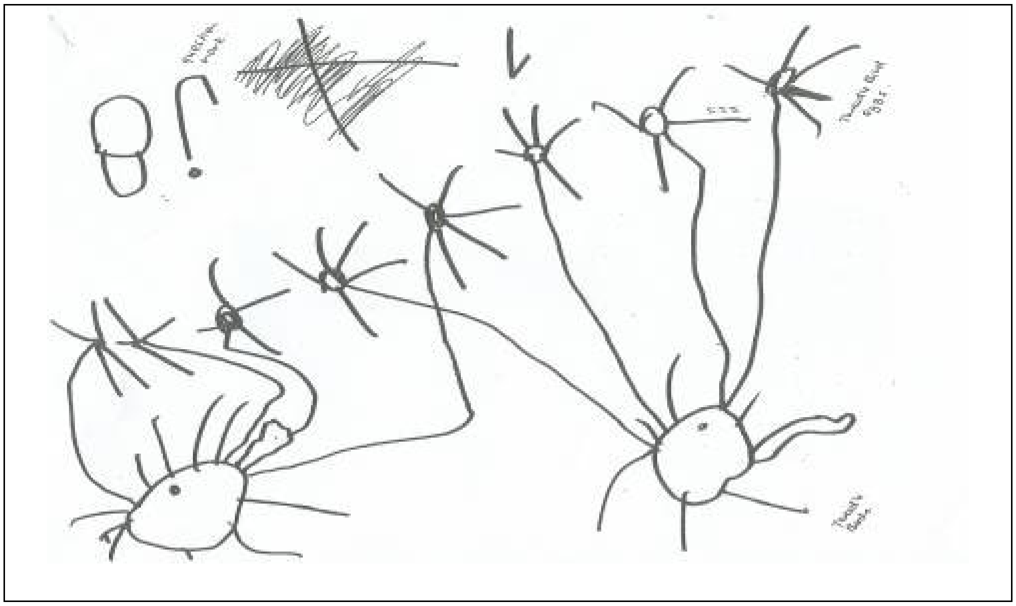


Figure 6: Kamrin’s ‘tweedle birds’

There is no advantage of ‘hurrying’ children into calculations. It is important to stress that children will need *many* opportunities to explore their own marks through self-initiated play within a rich learning environment, to allow understanding to develop.
This article includes only a brief summary of our findings and cannot reflect the full range of strategies children use (see Carruthers and Worthington, 2006): it seems likely that we will continue to revise and extend these categories in the future as our research reveals further evidence.

‘Signs’ of development
Children’s developing calculations include an impressive range of personal mathematical signs: some appear to be approximations of standard symbols, others are invented and some are implied within their calculation. In figure 7, Louisa combined drawings of strawberries, words and the numeral ‘6’ and then read her calculation, concluding, ‘… *and altogether there’s 6*’).

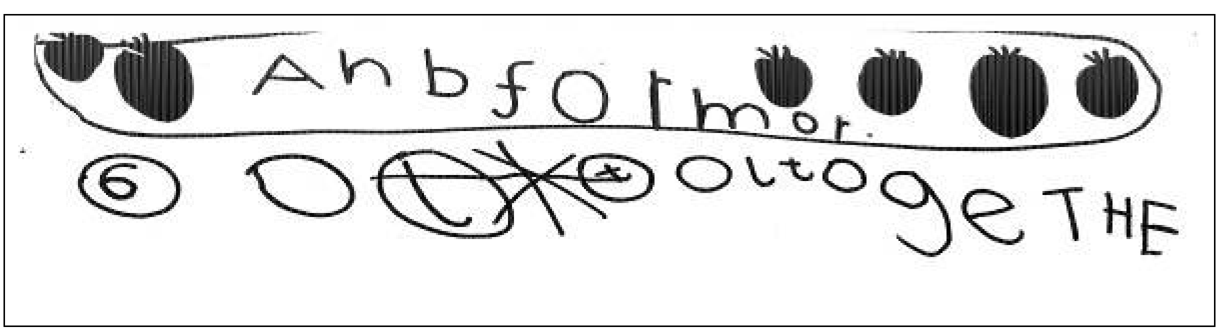


Figure 7: Louisa’s strawberries

Some strategies children use within their early calculations for addition and subtraction suggest an apparent *action* on the quantities involved. These include the use of arrows or hands to show that items have been added or subtracted and suggest that the physical action of adding or subtracting have become internalised (see figure 4). Children's own use of this sort of signs - which we term *narrative action* - is a significant indicator that they understand the operations and the function of operators within the calculations.

Children also often combine different features such as drawings with numerals, writing or standard symbols; or numerals with invented symbols. This is a self-chosen strategy known as 'code-switching' and is related to findings from research into second language learning, particularly on young children's *bi-literacy* in the Early Years. Children who are *bi-literate* are learning to write in two languages such as English and Spanish, or to write languages that have different alphabets such as English and Greek or Arabic.

Other strategies that children use in their mathematical graphics share similarities with bi-literate texts, including *approximation, invention, re-structuring* and *falling-back on their first written 'language'*: for these reasons we refer to young children's ability to combine informal and standard mathematical symbols as *bi-numerate*. .These strategies are important aspects of their development as they move between their own marks and representations and the standard written language of 'school' mathematics. It has previously been argued that the spoken and written language of mathematics is like a 'foreign' language for young children: some of the language used; the written symbols and algorithms are quite unlike anything that young children have met before, or are used differently in the children's home contexts and cultures.

Our evidence suggests that their own mathematical graphics allows them to 'bridge the gap' that Hughes identified, allowing children to 'translate' between their informal marks and understanding and standard written mathematics.

Teaching early 'written' mathematics

Although many aspects of practice have changed since the introduction of the *National Numeracy Strategy (*NNS) and the Foundation Stage curriculum, some uncertainties about the teaching of early 'written' mathematics persist and were revealed in a study we carried out with almost three hundred teachers throughout England.

With the introduction of the NNS there was an emphasis on children's own mental methods. The importance of children's informal written methods and the need to build develop links between mental and written strategies were also emphasised. We decided to explore the extent to which guidance on teaching written calculations (QCA, 1999a) had influenced teachers' thinking and practice.

From a study (in 2002) with almost three hundred teachers in the 3 - 7 year age-range, our findings revealed the continuing dominance of worksheets across all types of settings for children between the ages of 3 and 8 years, particularly in schools. Of equal concern was the rare occurrence of children's informal mathematical representations. Only a very small percentage of teachers said that they provided blank paper for children's mathematical marks or informal working and none kept the children's marks for their records or to help them understand of children's thinking. Sadly it appeared that young children's 'written' mathematics was only seen as significant when it was the outcome of an adult-planned activity.

Deepening understanding of children's mathematics

During the past sixteen years we have explored other aspects of children's learning including children's schemas, their early writing and drawings: studying these have helped us better understand children's mathematical development.

Our observations of children's schemas (Athey, 2007) helped us recognise that most schemas are mathematical. Collecting detailed observations enabled us to identify individual children's 'patterns' of (mathematical) behaviour' within their play that led to changes in our practice, and planning that drew on our observations of children's strengths and interests. It is encouraging to see that in the recent draft publication of the Early years Foundation stage (2006) that this significant aspect of young children's development (schemas) is now included.

Re-visiting our previous work on children's early writing, we looked at the children's explorations with symbols and the relationship between their early writing and their mathematical graphics. This revealed some close links between children's development in these two symbolic languages and point to effective practice for teachers and practitioners.

We also found a relationship between children's current schema interests and the marks children choose to make in drawing, writing or to support their mathematical thinking.

More recently we have looked at children's early beginnings with marks described by John Matthews in his exciting research on children's drawings (2003). Matthew's findings point to the same range of marks supporting all of children's drawing and mark-making. He identified what he terms 'generational marks' that arise through children's early gesture and movements. By closely examining the *forms* children use in their mathematical graphics and in their early writing it became apparent that the same *generational marks* underpin all children's visual representations and written symbolic languages. We have also explored the *purposes* and the *power* of drawings and how these relate to children's own mathematical representations (Worthington & Carruthers, 2005).

Together, our studies of children's schemas, their early writing and drawing development – supported by published research in these areas - combined to inform our understanding of children's mathematical graphics.

Symbolic languages

Symbolic languages include speech, counting, algebra, writing, diagrams, maps and systems such as road signs. The subject of mathematics has been described as 'really a problem-solving activity with symbolic tools', (van Oers, 2001, p. 63).

Young children will be familiar with some aspects of writing and numerals before they enter the Foundation stage, but it will be in school that they first begin explore the formal systems of writing and 'written' mathematics. The challenge for teachers and practitioners in the Early Years is to find effective ways of supporting young children as they learn these 'languages'. As Pound emphasises:

> '…learning to think mathematically is within the grasp of us all. Those of us who have the privilege of working with young children and their families are part of the solution to the problem. We must, as the pioneers of early childhood education, trust our insights and intuitions… we must also make our voices heard' (Pound, 2006, p.153).

Influences on learning

Children learn through their social and cultural contexts, communities and cultures and these influences shape their understanding. Socio-culturalism is now a widely accepted theory of learning and underpinned by a great deal of research. It points to the importance of the cultural contexts we create in early childhood and Key stage 1 settings.

Significantly it provides a perspective of children as powerful meaning-makers who are able to invent and adapt their own marks and symbols: they can use these to explore, represent and communicate their mathematical thinking. Together with cognitive (and affective) development and language, their representations assist this process.

Whilst the Foundation stage curriculum emphasises that 'Early Years experiences builds on what children already know and can do' (QCA, 2000, p.11), a number of studies have shown that young children develop considerable informal knowledge of mathematics at home (e.g. Aubrey, 1997) and through their informal marks, although this is not always recognised. This is where children's understanding of written mathematics really needs to begin - with their own marks and drawings they make at home and with what they already understand (informally) about mathematics before they enter nursery or school.

Creativity and mathematics

With the introduction of the Foundation Stage curriculum, came a renewed interest in 'creativity'. We had seen how very creative the *processes* of learning mathematics can be for young children and in 2003/04 we carried out another study with over 200 teachers and practitioners in the 3 – 7 year age-range. In this we wanted to explore teachers' and practitioners' ideas about creativity in mathematics and the extent to which guidance was influencing children's experiences.

- Practitioners appeared to see creativity in mathematics as specific resources, activities or *products* (e.g. role play and art) rather than as creative learning *processes* such as thinking and problem solving
- Only one practitioner was able to provide an example of a child doing something that they regarded as creative in mathematics, suggesting teachers and practitioners found it difficult to 'see' mathematics when observing children
- Self-initiated play, talk and thinking were seen as creative by only 9% of practitioners,
- Just 5% per cent of practitioners referred to children's mathematical mark-making as creative
- Children had very few opportunities to use their own marks and as in our previous study, practitioners said that they did not keep children's own mathematical marks
- Significantly, almost half of those who completed questionnaires said that they were either unsure or confused by the guidance for teaching early written mathematics.

Part of the confusion appeared to stem from many practitioners belief that mathematics in the Foundation stage should *only* be practical, although this is not what is said. When coupled with confusion about when to introduce written calculations, the guidance appears to have been misinterpreted as 'nothing mathematical should be written' in this phase. However it is clear from the research on children's early writing, from Hughes's findings and our own research that 'practical' mathematics alone is not sufficient to help children learn about the *written* language of mathematics.

Without good examples of what they might expect children's informal mathematics to look like it must be very difficult for teachers and practitioners to understand and interpret children's informal marks. Lacking support on the development of early 'written' mathematics also means that it is almost impossible to evaluate the marks they make

Official guidance on teaching early 'written' mathematics

The *Curriculum Guidance for the Foundation Stage* (CGFS) followed publication of the NNS; emphasising that teachers and practitioners ask children to "put something on paper"' (QCA, 2000, pp 71). However, as our research with teachers and practitioners showed, uncertainty remained about how they should teach early ‘written’ mathematics. For some this was interpreted as a need to ensure that written mathematics should ‘look like’ mathematics with numerals written correctly and neatly. Guidance for teachers on teaching ‘mental calculations strategies’ (QCA, 1999a) and on ‘teaching written calculations’ (QCA, 1999b) followed the introduction of the *National Numeracy Strategy*. Both publications support flexible and individual ways of thinking about numbers, calculations and problem solving, cautioning ‘at first children’s recordings may not be easy for someone else to interpret, but they form an important stage in developing fluency (QCA, 1999b, p. 12).

In the draft of the Early Years Foundation Stage (EYFS), mathematics is included under the heading of ‘problem solving, reasoning and numeracy’, setting out expectations for young mathematics within the play-based EYFS curriculum (DfES, 2006). It in interesting to see that within *communication, language and literacy*, the ‘writing’ section provides detailed guidance of children’s mark-making. We hope that in its final form (2008) the mathematics section of the EYFS will give greater emphasis to children making and communicating mathematical meanings through their on marks for early ‘written’ mathematics.

It is clear to us that unless young children have opportunities to build on their existing understanding when they enter the Reception class, there will remain a huge gulf between their informal marks and their understanding of standard written mathematical symbols and calculations. If we can also develop the sort of culture within our settings where children know that their own ideas and strengths are valued they will be able to make personal sense of the confusing business of standard written mathematics.

Pedagogy and children’s mathematical graphics

Children’s mathematical graphics are inclusive. Not only do their own representations allow children to make personal sense of the symbols and written language of mathematics, but they show how children’s thinking often to exceeds adults’ expectations, particularly in respect of *using and applying mathematics*.

It was clear how the pedagogical changes we had made concerning the learning environment supported children’s mathematical mark-making within play. We had also developed ways our adult-led mathematical activities to provide open opportunities for children to use their informal representations. However questions remained relating to pedagogy.

Modelling mathematics

We have always been very clear that supporting children’s mathematical graphics is not a ‘laissez-faire’ approach where children make careless marks or where no direct teaching is involved. An aspect that was at first more difficult to explain was the way in which we introduced standard symbols and horizontal calculations.

This led to two pieces of research in which I explored teacher modelling and to changes in the way in which we use *direct* adult modelling (below). Focusing on direct modelling revealed that when an adult models something at the *beginning* of a group activity or a maths lesson, almost all of the children will copy what the adult does – even if this was not intended. I found that by moving to modelling different ways of representing mathematics during ‘spare’ short sessions of time (around ten minutes) throughout the week, that this had the effect of adding to children’s ‘mental tool-boxes’ and works as an expending mental resource that they can draw on as they need. This is effective provided that this direct modelling is not then followed by a mathematics lesson.

From this we identified the following ways in which modelling supports ‘written’ mathematics:

- **Children’s socio-cultural contexts** at home and in their community, where they see written mathematical symbols displayed and used

- **Peer modelling,** when children see the various ways in which other children explore and represent their mathematical meanings

- **Indirect adult modellin**g: adults sometimes join in and represent something mathematical in the context of their play. Adults also provide mathematical print and notices in the learning environment

- **Direct adult modelling:** the adult provides models that offer new and alternative ways of representing mathematics (including standard symbols and calculations). The emphasis will be on using ways to represent and methods that are increasingly effective and provide meaningful contexts.

For further detail on modelling, see Carruthers and Worthington, 2006. p. 213-215.

Recent and on-going research
In 2003 we established a project for teachers and practitioners from ‘Early Excellence Centres’ (the forerunners of *Children’s Centres*) throughout England. This was an action-research project that we jointly managed and facilitated, and was funded by the DfES and the General Teaching Council. Teachers and practitioners developed their understanding of children’s mathematical graphics whilst developing their pedagogy and through collaborating in focused e-learning discussions. The outcome of this research was highly successful for both practitioners’ practice and for their children’s mathematics (Worthington, 2005).

More recently we have been supporting a two year study with a nursery mathematics ‘learning network’ of six Cambridge nurseries (Carruthers, *in progress*). The teachers and practitioners have become increasing aware of children’s early mark-making and have made a number of changes in their practice to support children’s mathematical marks. The findings from this project to date, show that practitioners’ involvement in this research has also raised the status of mathematics in their settings.

I am now engaged in research for my doctorate and gathering data in England and the Netherlands, on multi-modality, meanings and chidlren’s mathematical graphics. Elizabeth is now engaged in further research into the pedagogy of children’s early mathematical graphics within Children’s Centres.

Multi-modality
At the heart of children’s own mathematical graphics is one if its greatest strengths –the strategies they use as they explore their thinking in ways that are personally meaningful - making meanings with marks. Play provides particularly rich contexts for children to make *multi-modal* meanings as the rich examples provided by Kress, (1997) and Pahl, (1999) show.

‘Multi-modality’ means many modes or forms; many different ways of representing meaning and Kress’s work has revealed the complex ways in which cultures, new technologies and media influence different ‘texts’ that children create. The research on multi-modality is also influencing the teaching of literacy and children’s writing (e.g. Kress, 1997; QCA, 2004). The research I have recently started on children's *multi-modal* mathematical 'texts' is beginning to reveal new insights into children's mathematics.

Conclusion

And the answer to Hughes's question: 'yes' – provided young children *really* are encouraged to use their own mathematical representations and make their own meanings. Children's mathematical graphics point to a new vision of teaching early 'written' mathematics for children from 3 – 8 years. It reveals the amazing capacity that young children have to make sense of standard symbolic languages such as writing and mathematics, provided they can build on their informal 'home' marks and meanings. We continue to be excited by our research into children's mathematical graphics and the power of their visual representations.

Finding a personal 'passion' in education can give teachers a real sense of professional ownership at a time when they have had to adjust to a great deal of official 'being told what to do' and how to do it. Choosing to engage in professional reading and small-scale research projects in your setting or class can be an important part of this.

Our own research into children's mathematical graphics grew from small beginnings because as teachers we wanted help children better understand the beginnings of written mathematics. As the examples of children's mathematical graphics reveal, young children really are 'exceptional'.

References

Athey, C. (2007) *Extending Thought in Young Children*, (2nd. ed.) London: Paul Chapman,

Aubrey, C. (1997) 'Children's early learning of number in school and out' in I. Thompson (ed.) *Teaching and Learning Early Number,* Buckingham: Open University press.

Carruthers, E. (unpublished) 'The *Cambridge Nursery Mathematics Learning Network* 2005 – 2007': pedagogical approaches that support children's mathematical approaches'.

Carruthers, E. and Worthington, M. (2005) 'Making sense of mathematical graphics: the development of understanding abstract symbolism', *European Early Childhood Education Research Association* Journal (EECERA), 13, (1): 57-79.

Carruthers, E. and Worthington, M. (2006) *Children's Mathematics: Making Marks, Making Meaning.* London: Sage Publications, (2nd. Ed.).

Clay, M. (1975). *What did I Write?* London: Heinemann

DfES (2006) *Early Years Foundation Stage (EYFS) – consultation*: http://www.dfes.gov.uk/consultations/conResults.cfm?consultationId=1393

Gelman, R. and Gallistel, C. (1978) *The Child's Understanding of Number*, Cambridge MA: Harvard University Press.

Ginsburg, H. (1989) *Children's Arithmetic*, Austin. Texas: Pro-ed, (2nd ed.).

Hughes, M. (1986) *Children and Number: Difficulties in Learning Mathematics,* Buckingham: Open University Press.

Kress, G. (1997) *Before Writing: Re-thinking the Paths to Literacy*, London: Routledge.

Matthews, J. (2003) *Drawing and Painting: Children and Visual Representation,* (2nd ed.) London: Paul Chapman.

Nunes, T. and Bryant, P. (1996) *Children doing Mathematics*, Oxford: Blackwell.

Pahl, K. (1999) *Transformations: Meaning Making in Nursery Education,* Stoke-on-Trent: Trentham Books.

Papert, S. (1993) *Mindstorms: Children, Computers and Powerful Ideas*, London: Harvester Wheatsheaf, (2nd Ed).

Pound, L. (2006) Supporting Mathematical Development in the Early Years, Maidenhead: Open University Press, (2nd. ed.).

QCA (1999a) *Teaching Mental Calculations,* London, QCA

QCA, (1999b) *Teaching Written Calculations*, London: QCA

QCA (2000) *Curriculum Guidance for the Foundation Stage*, London: QCA.

QCA/UKLA (2004) *more than Words: Multimodal Texts in the Classroom*, London: QCA publications.

Sophian, C. (1996) *Children's Numbers*, Boulder, Colorado: Westview Press.

Oers, van B. (2001) 'Educational forms of initiation in mathematical culture' in, *Educational Studies in Mathematics,* **46**: 59-85,

Worthington, M. (2005) 'Issues of collaboration and co-construction within an online discussion forum: information ecology for Continuing Professional Development ', in *Reflecting Education,* Vol. 1, No 1-2, September 2005, pp 76-98. http://www.childrens-mathematics.net/publications_ejournal.pdf

Worthington, M. and Carruthers, E. (2005) 'The art of children's mathematics: the power of visual representation': Paper presented at Roehampton's '*Art in Early Childhood'* Conference.

The Children's Mathematics Network: www.childrens-mathematics.net