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The Art of Children's Mathematics: the Power of Visual Representation

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Abstract

In this paper we explore the relationship between children's early drawing and their 'mathematical graphics': *we originated this term to describe the range of marks children make though visual representation when exploring their mathematical thinking.*

Since the early 1990s we have explored more open ways of working with children as they represent mathematics. Our research was an evidence-based, ethnographic study with data collected over 12 years. We analysed 700 examples of children' own mathematical graphics from 3 – 8 year olds in homes, nursery classes and schools and developed a taxonomy tracing development from their earliest marks, drawings and symbols. This is the first time that this has been done (Worthington & Carruthers, 2003a). Indeed, whilst there is abundant research on early art (e.g., Matthews, 1999 & 2003; Anning & Ring, 2004; Malchiodi, 1998) there has been a dearth of research into the development of children's visual representations in mathematics and none that explores the relationship between children's drawings and their visual representations in mathematics: it is this aspect that we explore in this paper.

Our research shows how these representations are inventive, creative and joyous. They allow children to explore their thinking in ways that mirror the behaviours of artists and mathematicians. Viewing children's representations from a positive perspective allows children to explore and make decisions that support their development and has the power to support deep levels of cognitive demand and high levels of creativity.

Key words: visual representation, mathematical graphics, creativity, involvement, cognition

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Introduction

We have explored children's own mathematical graphics as they explore their thinking and develop their understanding of 'written' mathematics (Worthington & Carruthers, 2003a, Carruthers & Worthington, 2005). In this paper we focus on the ways that visual representations support learning and examine some examples of children's mathematical graphics to see the structures within them. Drawing is not only for 'art' or to illustrate a story: we argue that visual representation plays a critical role in children's learning of mathematics.

Child-initiated drawing - involvement and learning

In their study of levels of cognitive challenge in nursery and pre-school settings, Sylva et al analysed children's engagement in a range of activities (1986). They described as *high levels of cognitive challenge* when the child's activity was 'novel, creative, imaginative, productive, cognitively complex, involving the combination of several elements… is deeply engrossed' (p.60). '*Ordinary' levels* included 'child's activity is familiar, routine, stereotyped, repetitive, unproductive… not involving the combining of elements' and 'not engrossed' (p. 60).

One of the outcomes of their research was that child-initiated art was rated as having almost the highest level of cognitive challenge in which children engaged (Sylva et al, 1986). Using the same rating system in a study of a combined Reception and Y1 class (Worthington, 1996) I found that art achieved the highest level of 'cognitive challenge'.

Laevers focused on 'intrinsically motivated involvement' which he found to be one of the key indicators of quality learning outcomes (1993). This was subsequently developed by Pascal and Bertram (1997) as the *Effective Early Learning* project to support pedagogy. The evidence from these four research projects point to a clear relationship between levels of involvement and cognitive challenge, and to deep levels of learning linked with child-initiated art.

Athey's important work on children's schemas also shows the significant role that drawings have in children's development (1990) and in our own research on children's schemas (Worthington & Caruthers 2003a) we found that deep levels of learning and involvement were strongly linked. These studies suggest that the role of visual representation in children's learning merits deeper understanding and recognition within early childhood education.

Literature Review

1. Children's visual representations

A young child's first marks – sometimes referred to as 'scribbles' - are a major development in her step towards multi-dimensional representations of her world. Matthews argues that before the advent of speech, many infants 'form in visual media a powerful expressive and communicative language' that is not recognised by many as being significant (Matthews, 1999, p. 29).

Interest in children's early representations has a long history and has been researched for different purposes. Burt's work on intelligence tests in (1921) lead to Jane Goodenough's

(1926) ‘draw of a man’ tests that were used to help determine children’s mental ages, whilst figure drawing has dominated in other research, for example Koppitz (1968) and Cox and Parkin (1986). Others, such as Lowenfeld (1947) and Gardner (1980) make strong links between children’s drawings and their cognitive development.

Many of the studies on early drawings refer to scribbles and early drawings in an almost scientific way (for example, Burt, cited in Selleck, 1997). Kellog (1969) studied the ‘universal images that are express by children through similar formal structures’ Malchiodi, 1998, p. 15). Some researchers address scribbles as something that is useful for later drawing. Fein, (1997) elaborated what she terms the ‘visual vocabulary’ to describes children’s early marks, whilst Engle (1995) focuses on descriptions that stress meaning and Eng (1999) looked at changes in content.

Matthews argues that for many people, understanding of the development of children’s drawings has been ‘heavily influenced’ by the work of Piaget and of Luquet (1927); (Matthews 1999, p. 84). Piaget’s work has undoubtedly influenced teachers’ views of young children’s drawings, with those that are scribble-like often being valued less than those that became termed ‘visual realism’: the more a drawing ‘looked like’ what the child was representing, the more it was seen as representing the child’s maturity. Unfortunately the work of Piaget and Luquet became reduced to what has been termed ‘stage theory’ which has dominated understanding of children’s visual representation for many years. Matthews believes a great deal of research since this work ‘fails to question its underlying assumptions’ (p. 85). Stage theory has also led to a view of the ‘deficit model’.

Read (1943); Clegg, (1980) & Lowenfield (1947) ‘emphasised the importance of aesthetic and creative aspects of the curriculum’ (Anning & Ring, 2004). Others have focused on the way in which therapists use children’s drawings to help understand children’s troubled thoughts (e.g. Malchiodi, 1998).

In more recent years Kress (1997) and Pahl (1999) have explored wider aspects of visual representation through their work on multi-modality. Anning and Ring’s recent study (2004) explores case studies of children at home, pre-school and school and the impact of these different cultures on the children’s drawing. Matthew’s significant longitudinal research with his own children, on their development of drawing and painting exemplify the depth of children’s thinking and the significance of their early drawing (1999 & 2003). Of all of these studies we find the greatest relevance in Matthews work for our own.

2: Children’s mathematical graphics

When in the early 1990’s we began to explore mathematical graphics there was very little published on this aspect of teaching and learning compared to the wealth of books and articles on children’s early writing. Only one text explored this question in depth: in his study *Children and Number: difficulties in learning mathematics* (1986), Hughes highlighted the gap that exists between children’s early informal symbols and the abstract symbolism and language of school mathematics. He demonstrated how three and four year-olds could represent numerals in personal ways and which they could later 'read'. The examples Hughes provides (1986) a range of mathematical representations that he categorised as *idiosyncratic, pictographic, iconic* and *symbolic.* What was especially significant about this research was that the representations were the children's own: they show original and inventive responses to a range of questions. For the first time teachers would see something of the power of children's mathematical thinking, visually represented. However, Hughes' influence on teaching has sadly been sparse. In practice, whilst early marks may sometimes be valued as drawing and the beginning of writing, early mathematical graphics are rarely acknowledged (Matthews, 1999).

Atkinson's' publication (1992) includes a range of interesting and rich examples from children in chapters by teachers, and by academics. Others who support young children's own visual representations in mathematics include Whitin et al (1990); Whitebread (1995); Williams (1997); Gifford (1997); Pound (1999); Ewers-Rogers & Cowan (1996) and Vandersteen (2002), although these publications include few children's examples.

Thus there has been a small but growing interest in what has variously been termed 'emergent mathematics', 'mathematical literacy'; 'mathematics with reason' and what we term *mathematical graphics.*

The guidance on 'recording'

The growth of this literature was a departure from seeing children's 'written' mathematics as *recording,* which implies that children carry out a practical task and subsequently *record* their actions or the process of the calculation that they have just carried out mentally. When they record the expectation is that they follow the teacher's guidance, usually limited to one of a small range of forms such as "draw a picture of …"; "use tallies to show …" or "write how many…": children either colour in or record answers on printed worksheets or complete pages of 'sums' that the teacher has written (on the board or in the children's books).

A departure from recording (as copying and completing, or colouring in) came with the advent of the Numeracy strategy in England, in the guidance on teaching 'teaching written calculations' (QCA, 1999). This guide and related training on teaching 'jottings' and written methods has particularly influenced teachers of children of seven years and above.

Although the guide on written calculations uses the terms 'recordings', 'records' and 'jottings', it does make some useful points:

- 'At first, children's own recordings may not be easy for someone else to interpret, but they form an important stage in developing fluency' (p. 12)
- 'Children will need to have plenty of experience of using their own individual ways of recording addition and subtraction activities before they begin to record more formally' (p. 19)

However, regrettably it continues to describe ‘simple pictorial representations of what they have done practically’. The section for teachers of younger children (at the time of its publication, this included four and five year olds in Reception) is brief and offers no guidance on how teachers might understand and support young children’s own *recordings*. Only two examples are given and these are stylised pictures of objects counted with the calculations added on the same page, something we have very rarely seen and which suggests to us adult guidance was given on how to and what to ‘record’.

In England worksheets are also used throughout the Foundation stage with varying levels of use according to the type of Early Years setting (Worthington & Carruthers, 2003a, Worthington & Carruthers, 2003b). By the time they reach Reception classes, 89% of teachers use them and this rises to 100% for children in mixed Reception / Y1 classes and above (2003a. p. 6). Without opportunities to explore their own mathematical thinking in their own ways and through their own choices of visual representation, children are prevented from developing their own understanding of the written symbols of mathematics. Our evidence is clear: recording and worksheets prevent creative responses and fail to support children’s thinking at a deep level. They may ‘look like’ mathematics since there will be things to count, colour in or add, but we argue that they are to mathematics that ‘painting by numbers’ is to art.

In the Foundation stage curriculum (QCA, 2000) some of the strongest points that relate to visual representation and communication are found in the ‘creativity’ and ‘communications, language and literacy’ sections. It is unfortunate too that ‘creativity’ is presented as a separate curriculum area that relates only to art, dance, stories and music since it diminishes the importance of creativity throughout the curriculum, and fails to acknowledge the importance of creative thinking. There is no reference in the mathematics section to children’s earliest marks (scribbles) or their early visual representation that precede calculations. Training materials for children of 3 – 8 years also fail to acknowledge the tremendous potential of visual representations in the wider curriculum or for their mathematical development.

Representation

In contrast to *recording* we use the term *representing* since our emphasis is on children exploring their mathematical thinking through visual representation. This relates well to the numeracy strategy’s promotion of ‘mental models’.

Comments made by artists and mathematicians who find drawing valuable for their work strongly attest to the thesis that visual representation supports creative thinking, communication and learning (this paper, pp. 6-7). Visual representation by artists is not used as a *record* but as *representations* of their thinking and of embryonic ideas: they fulfil many purposes that are equally relevant to our youngest mathematicians in our classrooms.

Just as a narrow approach to the teaching of art limits creativity (Matthews, 1999 & 2003; Anning & Ring, 2004) a standard approach to ‘written’ mathematics for young children prevents them from working like artists or mathematicians. Of all the curriculum areas, mathematics has been viewed as an ‘absolute’ discipline with irrefutable logic and ways of working that are either ‘right’ or ‘wrong’. However, Ernest proposes that this view is ‘an idealization, a myth’ (Ernest, 199. p. 21). A fallibilist perspective therefore opens the door to different possible ways of working, and of representing mathematics for young children.

The thread that runs through some of the recent literature on children’s visual representation (e.g. Malchiodi, 1998; Matthews, 1999; Anning & Ring, 2004) and of children’s emergent writing and mathematical graphics is of the child’s ‘voice’, and of children making choices and decisions as they make meaning through visual representations. The official literature does not always reflect this and may be due in part to the absence of a research-base for some of the official publications for teachers through government ‘initiatives’ that are sometimes hurried into the classroom. There is an urgent need for teachers to understand children’s development of visual representation.

Visual representation

Against this background of research and curriculum expectations we explore the relationships between visual representation in early childhood and adulthood, and children’s mathematical graphics.

The first aspect we consider is the purpose for which visual representations are used by adults and children. Secondly we explore the ways in which the purposes of drawings and the structures children use within their drawings help them to develop, explore and communicate meaning. Drawing on the work of *Power Drawing* (Adams, 2002) and Matthews (1999 & 2003) has allowed us to look deeper at the ways in which children’s marks and representations develop.

Through the lens of an artist

Purposes for drawing

In their selection of works for the Royal Academy’s Summer Exhibition (2004), Allen Jones and David Hockney mounted a special gallery of drawings with contributions from ‘people who are not artists, but who use drawings in their work’ (R.A. catalogue. 2004).

Exhibits selected included work from individuals who are eminent in their field: designers; a poet; musicians; the coach of the England Rugby team; a theatre designer; zoologist; film maker; surgeon; chemist; scientists – and several mathematicians. Each exhibit was notable for the way in which the sketch, notes, diagram, plan or drawing – or combination of these – conveyed meaning in a distinct way. Some explained that they drew: *‘to visualise processes’* (Eno, musician and composer); in order to *‘be more specific in a different way to words’* (Clark, dancer and choreographer); ‘*as working documents’* (Squire, former RAF pilot) for *‘exploring ideas’* (Hunt, structural engineer) and as *‘free responses’* (Birtwhistle, composer), (R.A. catalogue, 2004). And, as we shall see, the mathematicians who exhibited in the show also described some important aspects of the role of drawing in the work

Mathematical drawings

One of Britain's finest mathematicians and physicists of the past fifty years, Roger Penrose explained that drawing is valuable in his work on a number of levels: '*it can be used for expositional purposes, in the form of diagrams. It helps thinking by allowing me to visualise what's going on. It can be used as a form of notation, and, on a less abstract level, it can work simply by representing exactly what it stands for*.'(Penrose, in Highfield, 2005). At the launch of the Campaign for Drawing, Penrose revealed how '*doodles'* helped him' *to wrestle with highly abstract calculations'* (Highfield, 2005).

At the Royal Academy's 2004 exhibition, Penrose described his use of drawings *to represent complex aspects* of the universe and for *recording thoughts* whilst Cheng, another important mathematician who uses visual representation in his work, explained that *though this struggle, a beautiful structure emerges* (R.A. exhibition catalogue, 2004).

We propose that these processes (described here in italics) are similar to those in which young children engage when exploring their mathematical thinking through their graphics.

Evidence of the power of drawing

In 2000 the Campaign for Drawing was launched in England and has published 'overwhelming evidence of drawing's value in teaching and learning'. Its education programme 'Drawing Power' is seen as 'an original and timely initiative, which focuses attention on the neglected, and still little understood areas of visual intelligence' (Adams, 2001). The Power Drawing 'notebooks' list three 'purposes of drawing': *perception, communication* and *invention*; lens through which all visual representations may be viewed (Kress, 1997; Pahl, 1999a & 1999b).

Perception, intuition and thought

Perception is described by Power Drawing as: 'drawing that assists the ordering of sensations, feelings, ideas and thoughts… to explore and develop observation and interpretative skills to investigate and understand the world' (Adams, 2003).

In ways that are still not fully understood drawing aids cognition and 'encourages and develops the connection between thinking and doing which must take place at intuitive as well as more consciously determinative levels.'(Rhodes, undated).

This power of drawing to support thinking is recognised by many artists. Avis Newman selected the drawings for 'The Stage of Drawing' exhibition (Tate Britain 2004) and regards drawing as 'the nearest equivalence to operations of thought' (2003b. p.7). Drawings, she believes also contain a 'deeply rooted history' and whilst early sketches may be seen as 'throwaway things, waste… yet they are essential to the articulation of thoughts and ideas as a way of seeing what one is thinking' (Newman, 2003b. p. 77).

However, children's earliest marks are often referred to in a somewhat dismissive way as 'scribbles' (Matthews, 2003, Worthington & Carruthers, 2003) or those that might appear to be 'idiosyncratic responses' (Hughes, 1986, p.57). Whilst we can only conjecture about the possible meanings of some early visual representations, we believe that children's early marks carry meaning and intentions. What may appear as 'throwaway things' have an essential role in supporting deep levels of thinking: each act of visual representation arises out of previous mark-making experiences and of influences within the individual's socio-cultural experiences and is therefore polyadic (Worthington, 2005).

Communication, language and meaning

Communication is seen as drawing 'that assists the process of making ideas, thoughts and feelings available to others… it is likely that certain codes or conventions will be used …to help understand what is being communicated' (Adams et al, 2004. p. 2). Drawn marks are often more powerful than words, which, according to Leonardo da Vinci are 'poor resources for capturing complexity' (Jardine, 2004. p.13).

Brown argues that 'when drawing is recognised as a meaning making process' it 'becomes central to the teaching and learning of young children' (Brown, 2003. p. 10). Drawing on Vygotsky that 'in meaning, answers to our questions about the relationship between thought and speech can be found' (Vygotsky, 1962, p. 5), Brookes proposes that for the child 'drawing mediates the experience' and 'facilitates higher-order thinking' (2003 pp.5, 8).

Young children's visual representations 'speak' to us about their thinking if only we are open to hearing the 'voice' of the child since 'drawing has the potential to reduce to its smallest, the gap between meaning and non-meaning…' with the marks of a drawing as 'already potentially a sign… even if we do not have the means to decode it' (Newman, 2003b, p. 100).

Invention

Invention is described as drawing that assists the creative manipulation and development of thought (when) ideas are at an embryonic stage… when the drawer experiences 'reflexive oscillation' between impulse, ideas and mark, receiving feedback from the marks appearing on the page, which prompt further thought and mark-making' (Adams et al, 2004. p. 2).

The term *reflexive oscillation* suggests too our model of 'bi-numeracy', (Worthington & Carruthers, 2003a. p.77). We believe that it is the combined information from the individual's marks and from socio-cultural contexts that provide this feedback.

Invention also allows 'ideas to be 'explored, repeated, refined … discarded (and) combined… alternatives are sought and a range of possibilities explored. Key activities here are translation, formation, transformation and invention', (Adams et al, 2004. p. 2). Children's mathematical graphics show clearly that these processes are at work, although they can only explore these behaviours when they are free to explore with marks and materials in their own ways and in their own time.

Mathematical Graphics

The study

The background to this paper is the central part of our extensive study (Carruthers & Worthington, 2005) in which we analysed children's own graphics from naturalistic contexts. Graphics analysed came from a range of settings including home, nursery classes and Reception and K.S.1 classes in schools, in the three to eight years age range. The samples analysed came from a diversity of cultural backgrounds and included children with special needs.

Methods and Approaches

This was a qualitative research study based on naturalistic enquiry methods which adopt an inductive approach. It was an ethnographic study (Hoshmand, 1989) which attempts to study a group in their naturalistic surroundings, in this case, children at home and in school. The study also drew upon phenomenology inquiry (Polkinghorne, 1989) which attempts to describe and elucidate human experience.

The context of the samples

The samples of the children's graphics were taken from our own teaching situations over a period of 12 years. They came from different teaching contexts from direct teaching to children's free play. We were influenced by the developmental literacy movement (e.g. Hall, 1987, Cambourne, 1988, Goodman, 1986) and their emphasis on environments that are alive with print and that see children's attempts at their own writing in a positive way. We describe the features of our pedagogy and of the learning environments we created in our study (Worthington & Carruthers, 2003a).

Since the data from our original study centred on the graphics from a mathematical perspective, we can only give an overview in this paper. As teachers, we were very familiar with children's development in writing but as we looked at the 700 samples of mathematical graphics and searched for patterns we constantly revisited our hypothesis, recognising that mathematical graphics are more diverse. This is perhaps so since the different areas of mathematics often suggest quite different graphical approaches. For example, in representing data young children may use pictures or ticks and move towards increasingly clear or standard layouts.

We classified the data into two distinctive groups; one we developed from Hughes's categories, of the kind of marks children chose to make. We also analysed the mathematics children used and developed a taxonomy that shows the different aspects of their mathematics, explored through their graphics (Worthington and Carruthers 2003a).

Analysing children's mathematical graphics from a drawing perspective

Although we saw the children's graphics as mathematical it became increasingly obvious to us that the children's graphics could also be termed 'art'. This was confirmed by several artists who looked at the children's mathematical graphics with interest: elements of what might be termed creativity, were very clearly recognised in these representations.

From a drawing perspective we firstly selected criteria based on Power Drawing, (Adams 2002). This was originated by Ruskin in 1871 (Glancey, 2002) who believed that that drawing was the foundation for visual thought. This meshes well with our understanding of children's mathematical graphics which helps the children think through the mathematics. Power Drawings' criteria for looking at the children's graphics from the broader perspective of visual representation are of *perception, communication* and *invention*.

We combined our prior use of the comparative method with analyses through Matthew's 'generational structures' and the features identified by Power Drawing to strengthen the validity of our findings.

Structures of visual representation

In contrast to the Power Drawing criteria, Matthews looks at the *structures* children use to express their thinking: this work is a departure from previous studies of children's visual representations in the detail through which Matthews explores and analyses children's actions and marks (1999). Matthews strongly supports the premise that children's marks are not haphazard scribbles but are products of a systematic investigation. He observes children's actions and mark-making through what he terms *first, second* and *third generation structures* and shows how these structures are found in subsequent drawings as children develop.

First generation structures refer to *horizontal and vertical arcs* and marks made by *push and pull* movements. These drawings use action usually large arm action. Matthews describes his own children from six months to two years using these structures.

The *second generation structures* are *continuous rotation*, *demarcated line-endings*, *travelling zig-zags*, *continuous lines* and *seriated displacements in time and space*.

Third generation structure is where the child organises together and transforms first and second generation structures. Third generation structures are *closure*, *inside /outside relations*, *core and radial*, *parallelism*, *collinearity*, *angular attachments*, *right-angular structures*, and *U shapes on baseline*. These structures support all visual representations.

Analysis of samples from a drawing perspective

We randomly selected six examples of mathematical graphics from three different age-groups (nursery, reception and KS1) to discuss from a drawing perspective (see figures 1 - 6).

Figure 1: Molly, 3 years, 11 months: "seven, six and number eight"

Molly chose to select some paper in the graphics area and made a line of letter and number like marks at the top of the paper: beneath this the marks are different but still look like writing. Molly's marks show *perception* and assisted the ordering of her 'feelings, ideas and thoughts' (Adams, 2003, p.2): She appears to have had an idea of what numbers looked like and used all her knowledge to produce these marks.

I*nventiveness* is a striking feature of Alex's numbers (figure 2, below), where 'translation, formation, transformation and invention' are key features (Adams, 2003, p.2). Like many young children, he has appropriated the first letter of his name to stand for something he wants to say (here it is for '2'). This self-initiated exercise combined some elements of his number knowledge. He has generated symbols that show a consistent 'pattern' (e.g. his '6', '7' and '8'). His symbols for '5' show consistency and '4' shares some of the appearance of the standard symbol and thee is a 'three-ness' about his numeral '3'.

If we compare Alex's numbers to Molly's numbers we can see a marked difference. Molly has used second generation structures *demarcated line endings*, *seriated displacements*, and *travelling zig-zags* that bear a similarity to the physical action of writing.

Alex has used third generation structures of *closure*, *angular attachments*, *core and radial*, *outside/inside* and *U shapes on a baseline*. He has a sense of the code of written numbers and his own representations help him explore the way in which this 'works'.

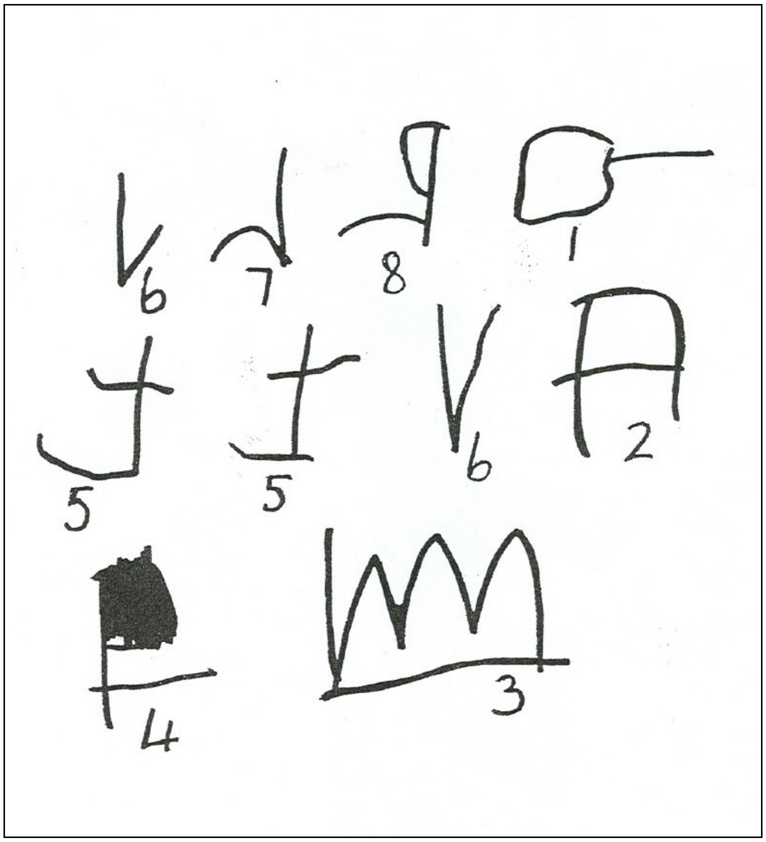

Figure 2: Alex's numbers (4 years, 11 months)

Kamrin's (figure 3) drew 'Tweedle birds' that combine several features using standard maths and invented symbols and drawing and was able to visualise this in his mind. Kamrin was using this to work out a division problem that he had and selected the quantity to divide. His *invented* 'Tweedle birds' helped him think through the mathematics: as Adams describes, his drawing is in fact, 'one of a series, where ideas are explored… refined… where alternatives are sought and possibilities explored' (Adams, 2003, p.2). His 'Tweedle birds' combine *core and radial* and *closure* structures, *U shapes on a baseline* and *angular attachments*.

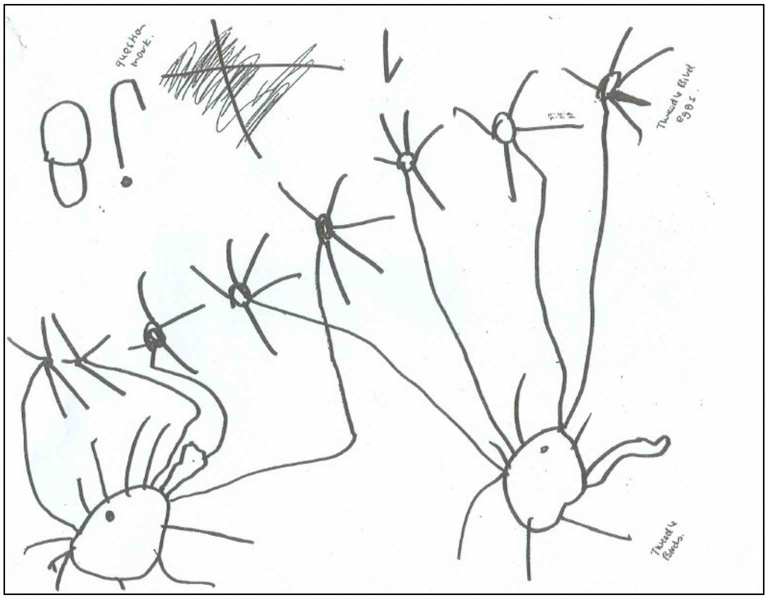

Figure 3: Kamrin's 'Tweedle birds' (5 years, 7 months)

Alison (figure 4) was multiplying using the 99 times table. Her thinking is very clearly documented here and like the artist has gone through several drafts and ideas, the first being using the iconic form of one stroke representing one. She realised after a while this was not going to work and returned to mental methods. Her *inventiveness* is seen in her writing of '298' as '20098' and in her ways of working. She was not afraid to try out new things and cross out and start again. Eventually she worked out'5 x 99'. Alison had never worked beyond 100 before and she needed to draw on all her previous knowledge to develop her ideas. She used 'certain codes or conventions' in her choice of symbols (Adams, 2003, p.1), to *communicate* her ideas to others.

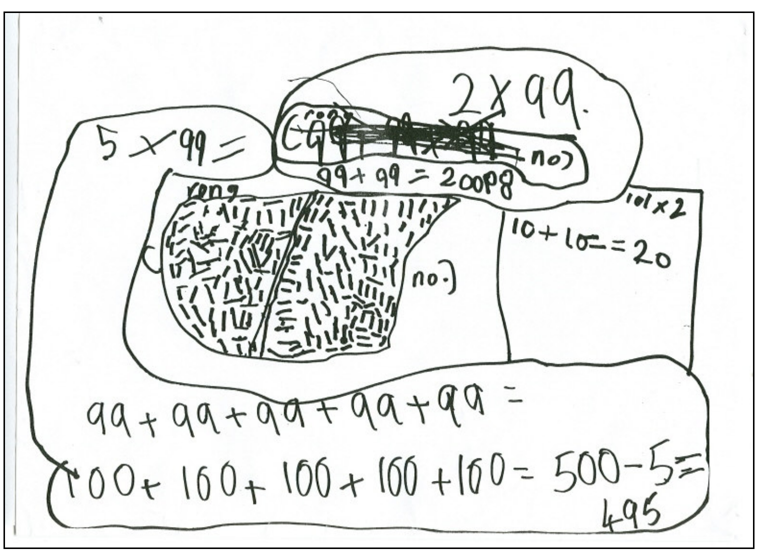

Figure 4: Alison – multiplying by 99 (7 years)

Alison's use of *closure* to manage space on her paper is interesting. Her numerals are combinations of all the generational structures that she has mastered and show that the complexity of producing standard written letters and numerals is a considerable achievement for young children.

Originality, freshness and inventiveness are distinct features of all the children's graphics. Amelie's example particularly (figure 5), combines all these elements which, in our category of forms we describe as *dynamic*. Amelie's representation was in response to a dice game she played. In respect of the mathematics she represented the dots on the dice (carefully counting out loud each time) and used the '4' of her age and the 'A' of her name to represent the numbers she 'read' on the dice each time. Other letters, numbers and mathematical symbols were drawn from what she knew and what she saw her peers write. The central zig-zag shape suggests the bouncing of the dice on the table top, though we cannot be cure that this was so.

The *dynamic* form is full of life and suggestive of action. Each graphic is unique and this is what separates it from standard mathematics where all children's responses are the same. Bentley (1925) who studied the structures of snowflakes closely reported 'every crystal was a masterpiece of design and no one design was ever repeated' (Bentley, 1925). Like snowflakes none of the children's representations are alike.

Amelie has included both second and third generation structures. Combining structures generate complex visual representations and support complex mathematical thinking. This example appears to show what Adams describes as *perception*, enabling her to 'explore and develop observation and interpretive skills to investigate and understand the world' (Adams, 2003, p.2).

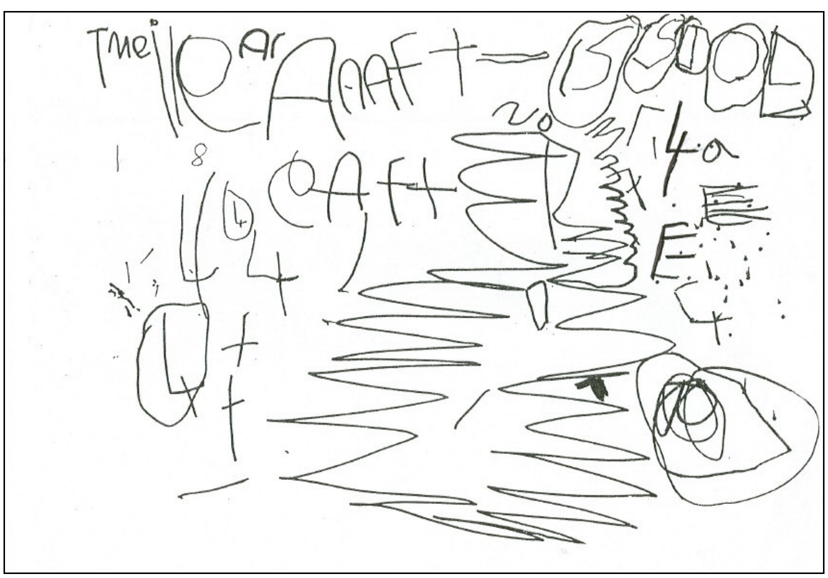

Figure 5: Amelie's dice game (4 years 7 months)

Elliot (figure 6) explores his thinking through *core and radial*, *closure* for his self-initiated mark-making of three objects. He has also drawn *continuous lines* though has not combined or generated new structures. Mark-making can be an inclusive activity within school systems and although less mature than his peers, Elliot has managed to express himself with his known structures. Perhaps if he has more opportunity and exposure to a wealth of mark-making then he will be able to develop thinking in more complex ways, through his visual representations.

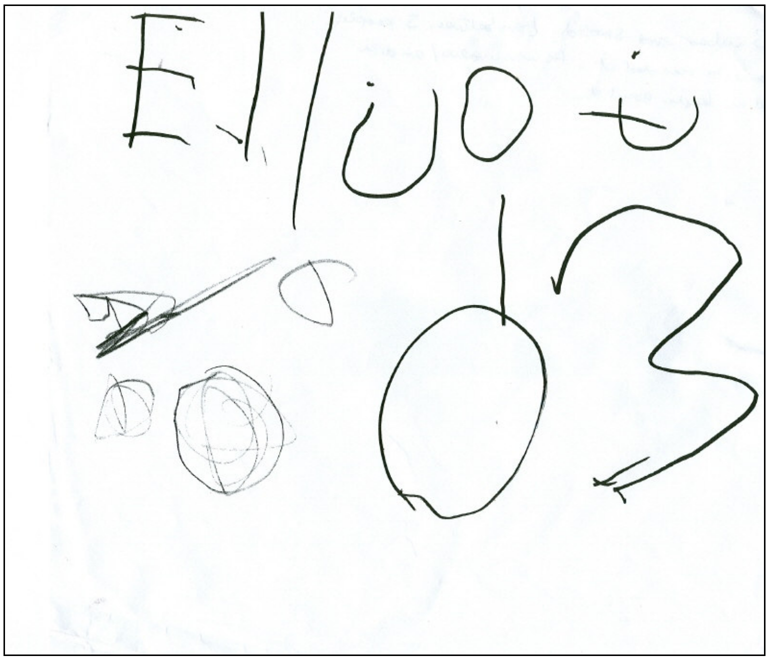

Figure 6: Elliot, sharing numbers (4 years. 11 months)

These powerful structures are in some respects similar to the forms Chris Athey found in children's drawings as they explored their schemas. We have also seen how children's schemas support their mathematical understanding and are evident in all their visual representations (Worthington & Carruthers, 2003a).

Commenting on our examples of children's mathematical graphics, Matthews emphasises that they 'show how children's mathematical thinking is embedded in certain visual structures (structures which they are finding in other media and within the environment). I describe very similar kinds of drawings and have always argued that they are rich in language and mathematics' (Matthews, 2005).

Conclusion

Looking at children's mathematical graphics from the wider perspective of visual representations, we have shown that *perception*, *communication* and *invention* are significant features of their mathematical graphics. Such a perspective supports a view of children as powerful and creative learners and helps reduce the false dichotomy between curriculum and 'subject' areas'. Matthew's powerful visual *structures* allows us to look more closely at the details of their representations and confirms that the synthesis of their marks move freely between drawing, early writing and mathematical graphics and other visual literacies.

Implications

Young children need many opportunities to explore, practice, repeat and refine their visual representations: they provide children the means to explore their mathematical thinking and are quite different to 'pencil control' or of copying letters and numbers.

The Numeracy Strategy emphasises that children's mental ability is much more advanced than their fine motor skills. Children's mathematical thinking is slowed if they must follow a narrow approach of *recording* 'written' mathematics. However our findings suggest that children's own mathematical graphics could keep abreast of their mental ability and have the power to support children in realising their mathematical potential.

Matthews emphasises that as children begin to draw and paint, they 'make an intellectual journey which has musical, linguistic, logical and mathematical as well as aesthetic aspects.' He argues that 'all these are endangered if we do not understand the development of drawing…The problem has been that… with some notable exceptions… few people have been able to see what that contribution actually is' (Matthews, 2003. p. 14). The art of children's mathematics shows clearly the possible 'intellectual journeys' that all children can travel.

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