[The abstract symbolic 'written' language of mathematics]

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1. What are the main difficulties with numeracy and mathematics in the Early Years?

The abstract symbolic 'written' language of mathematics is varied and complex (1) and is the area of mathematics that young children find most difficult (2, 3). Without suitable support for this essential aspect children will fail to develop their full potential in mathematics.

Mathematical skills are important, however, emphasising only 'basic skills' (2, 4) limits children's understanding and progression in mathematics and numeracy. In England OfSTED has repeatedly raised concerns, emphasising,

> It is of vital importance for pupils of all abilities to shift teaching and learning in mathematics away from a narrow emphasis on disparate skills towards a focus on pupils' mathematical understanding' (5: p.3).

2. Good practice

Research informed practice: the abstract symbolic language of mathematics

- Young children's understanding and confidence in numeracy and mathematics are supported by their ability to use signs and symbols flexibly (3, 6, 7), yet there is no official guidance on this.
- Evidence gathered through research throughout England during the past two decades (2) has focused on the educational concept of *children's mathematical graphics* in the birth – 8-year age range. Used in personally meaningful ways children's own representations gradually integrate standard signs of 'school' mathematics, highlighting their value for problem solving (2, 8, 9) and wider aspects of the curriculum. Research has also identified children's informal methods for calculations as of considerable benefit for children in Primary and Secondary schools (10, 11).
- The difference between 'recording' mathematics and *children's mathematical graphics* is one of quality and depth of thinking (12), children engage in creative thinking, reasoning, problem solving, negotiation and co-construction, using and applying their mathematical understanding.
- This educational concept supports numeracy and all aspects of the mathematics curriculum from birth – 8 years. It has considerable value in establishing strong and effective foundations for written mathematics in the Early Years, building mathematical fluency and confidence.
- Analysis of over 700 examples enabled children's learning trajectory to be charted, the first time that this has been done (8,13).

- This concept supports all children including those who are gifted or have specific learning needs, and no difference has been found in the willingness of boys or girls to represent their mathematical thinking.
- Teachers' and other Early Years professionals' understanding of effective approaches to *children's mathematical graphics* has had a positive impact on children's achievement.
- A major study of mathematics teaching in England gave prominence to *children's mathematical graphics*, making a number of recommendations (14). Subsequently Carruthers and Worthington were commissioned to write *a* publication on Early Years mathematics for teachers and Early Years professionals (15).

Doctoral research (1): *Maulfry Worthington: VU University, Amsterdam*

- Young children's learning begins at home (16, 17), and in early childhood play is acknowledged as the 'leading activity', providing a 'bridge' between spontaneous and scientific concepts (18, 19) and connecting informal representations with the formal written language of mathematics.
- Recent doctoral research was conducted through a longitudinal ethnographic study in a nursery in a Children's Centre cited in a large city in the southwest of England, in an area designated one of the 30% most deprived in England.
- The research focused on the emergence of children's understanding and use of early 'written' mathematics in various contexts, the findings providing evidence of a high incidence of mathematics within their spontaneous pretend play that spanned the breadth of the curriculum (20).
- A significant finding highlighted children's growth of *graphic symbols* between 3 – 4 years of age that underpin symbolic languages such as mathematics (21, 22).

Pedagogy

- The incidence, success and sustainability of *children's mathematical graphics* are dependent on the values, beliefs, knowledge and practices of headteachers and staff members in early childhood settings children confidently representing their mathematical thinking in their play, in other child-initiated contexts and in adult-led groups and classes (23, 24).
- *Children's mathematical graphics* have contributed to raised standards in mathematics in schools and authorities where this has been a focus of professional understanding.
- Effective pedagogy includes modelling informal and standard mathematical signs and texts for a range of authentic purposes in contexts meaningful to the children (2).

Doctoral research (2): *Elizabeth Carruthers: University of Bristol* [1]

- A participatory research study of teaching and learning in mathematics with a focus on children's own problem solving uncovered that reception class teachers were confused about how to introduce calculation and struggled with engaging children in their own problem solving (25).
- The teachers found it difficult, at first, to open up the mathematics and to understand children's own mathematical notation, signs and symbols (2; 8; 23). However, by the end of the study, the teachers had many narratives of how the children in their class were significantly more challenged in mathematics (26).
- Throughout the study these teachers expressed their concern of the restrictive pressures of the present system in creating a significant barrier to children's mathematical learning and achievement.
- From the findings of this study it appears there needs to be greater emphasis in mathematics teaching on listening to children's own mathematics through real contexts and children's own *mathematical graphics* that are open and child led.

3. What are the policy solutions that would help solve these problems?

This combined research highlights the need to move beyond 'basic skills' – especially regarding the abstract written symbolic language of mathematics: to this end we recommend:

- ✓ An increase in teachers with QTS in all nursery and pre-school settings.
- ✓ Greater clarity on the Foundation stage as a distinct phase, with appropriate mathematical experiences and high expectations.
- ✓ That there is an urgent need to address issues with Reception, acknowledging its position *within* the Early Years Foundation stage, and emphasising meaningful and challenging learning experiences for mathematics for children of 4 – 5 years of age.

Developing professional knowledge and expertise including:

- ✓ The 30 designated Mathematical Hubs in England are all Primary or Secondary led, and Early Years mathematics is not a focus. We therefore recommend a network of designated Early Years Mathematics hubs [2], by acknowledging the wealth of early educational excellence in maintained nursery schools and developing early mathematical hubs around the

[1] Elizabeth Carruthers is headteacher of Redcliffe Children's Centre, maintained nursery, National Teaching School and Research Base, Bristol.

[2] A major two-year study in England of mathematics continuing professional development (CPD) featured Carruthers and Worthington's initiative of local ***children's mathematics network learning groups*** (Early Years mathematics hubs), (27; 28; 29).

Nursery School Teaching Schools and other outstanding Nursery Schools (not to be mistaken for nursery classes in Primary Schools).

✓ To develop professional understanding of mathematics in the Early Years, we recommend development of Early Years Mathematics Masters modules - linked to EY Maths Specialists and EY Play Specialists [3].

References

1. Duval, R. 2006. A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, 61, 105-131.
2. Carruthers, E., and M. Worthington. 2006. *Children's mathematics: Making marks, making meaning*. 2nd ed. London: Sage Publications.
3. Hughes, M. 1986. *Children and number: difficulties in learning mathematics*. Oxford: Basil Blackwell.
4. Moyles, J. and M. Worthington. 2011. The Early Years Foundation Stage through the daily experiences of children. *TACTYC Occasional Pape*r no.1.
5. OfSTED. 2012. *Mathematics: Understanding the score*. London: Ofsted.
6. Gifford, S. 1997. 'When should they start doing sums?' A critical consideration of the emergent mathematics approach. In I. Thompson, Ed. *Teaching and learning early number*. Buckingham: Open University Press. First edition.
7. Whitebread, D. 1995. Emergent mathematics or how to help young children become confident mathematicians, In J. Anghileri (ed.) Children's mathematical thinking in the primary years. London: Cassell.
8. Carruthers, E., and M. Worthington. 2005. Making sense of mathematical graphics: the development of understanding mathematical symbolism. *European Early Childhood Education Research Association Journal* 13, no.1: 57–79.
9. Carruthers, E. and Worthington, M. 2008. Children's mathematical graphics: young children calculating for meaning. In I. Thompson, ed. *Teaching and Learning Early Number*, Maidenhead: Open University Press, 2nd Ed.
10. Terwel, J., van Oers, B., van Dijk, I., van den Eeden, P. (2009). Are representations to be provided or generated in primary mathematics education? *Educational Research and Evaluation*, 15(1), 25-44 (2009). doi: 10.1080/13803610802481265.
11. Pape, S. J., & Tchoshanov, M. A. (2001). The role of representation(s) in developing mathematical understanding. *Theory into Practice, 40*(2), 118-127 (2001).
12. Carruthers, E. and Worthington, M**.** 2009. Children's mathematical graphics: understanding the key concept, *Primary Mathematics, Autumn 2009,* Volume 13, Issue 3, 3-5.
13. Carruthers, E., & Worthington, M. 2013. Taxonomy charting children's mathematical graphics. http://www.childrens-mathematics.org.uk/taxonomy.pdf

[3] In 2013 *Redcliffe Children's Centre' Research Base* in Bristol established a successful double EY Mathematics Masters module: it is accredited by Bath Spa University, and repeated in 2014. Several teachers completing these Masters modules are now *Specialist Leaders in Education* (SLEs) for Early Years mathematics, supporting colleagues and teachers in other settings within their authority, and leading CPD.

14. DCSF. 2008. *The independent review of mathematics teaching in early years settings and primary schools.* London: DCSF. http://dera.ioe.ac.uk/8365/1/Williams%20Mathematics.pdf.
15. DCSF. 2009. *Children thinking mathematically: PSRN essential knowledge for early years practitioners.* London: DCSF. http://dera.ioe.ac.uk/743/1/c825856047856656f5d94e1406555f65.pdf.
16. Aubrey, C. 1997. *Mathematics teaching in the early years*. London: Falmer Press.
17. Carruthers, E. 1997. *Number: a developmental theory: a case study of a child from 20 to 44 months.* Unpublished Masters (M.Ed.) Dissertation. University of Plymouth.
18. Vygotsky, L.S. 1978. *Thought and language*. Cambridge, Massachusetts: MIT Press.
19. Vygotsky, L.S. 1986. *The collected works of L.S. Vygotsky. Volume 1: problems of general psychology*. New York: Plenum Press.
20. Worthington, M. and B. van Oers. 2014. Pretend play and the cultural foundations of mathematics, *European Early Childhood Education Research Journal.* 24(3), *forthcoming.*
21. Worthington, M. 2012. Children becoming expert symbol users. In McAteer, M. Ed. *Improving Primary Mathematics Teaching and Learning*. Maidenhead: OUP.
22. Worthington, M. and B. van Oers. 2015. Children's social literacy practices and the emergence of graphic symbols. *Submitted*
23. Carruthers, E. and Worthington, M. 2011. *Understanding children's mathematical graphics: beginnings in play*. Maidenhead: Open University Press.
24. Worthington, M. 2015. Mathematics and the ecology of pretend play. In J. Moyles, Ed. 4th Ed. *The Excellence of Play*. Maidenhead: Open University Press.
25. Carruthers, E. 2015. Listening to children's mathematics. In *Mathematical Transitions to School International Perspectives* Eds. B. Perry, B. Gervasoni and A. McDonald. Springer, Australia. *Forthcoming.*
26. Carruthers, E. 2012. Are the children thinking mathematically? The pedagogy of children's mathematical graphics. In McAteer, M. ed. *Improving Primary Mathematics Teaching and Learning*. Maidenhead: OUP.
27. NCETM. 2009. *Final report: Researching effective CPD in mathematics education (RECME).* National Centre for Excellence in Teaching Mathematics. https://www.ncetm.org.uk/files/387088/NCETM+RECME+Final+Report.pdf
28. Back, J. and Joubert, M. 2009. Reflecting on practice in early years' settings: developing teachers' understandings of children's early mathematics. *Proceedings of the British Society for Research into Learning Mathematics* 29(1) March 2009. http://www.bsrlm.org.uk/IPs/ip29-1/BSRLM-IP-29-1-03.pdf
29. Carruthers, E. and Worthington, M. 2009. An early years CPD initiative for mathematics: the power of collaborative, 'grass roots' learning. *Informal Proceedings of the British Society for Research into Learning Mathematics* 29(1) March 2009, (35 – 40). http://www.bsrlm.org.uk/IPs/ip29-1/BSRLM-IP-29-1-Full.pdf

Maulfry Worthington and *Elizabeth Carruthers* are co-directors of the international **Children's Mathematics Network**, a not-for-profit organisation with over 1000 members.

Its aims are to enrich children's mathematical understandings, to share

research and information and to promote effective practice in support of *children's mathematical graphics* - with all those engaged in early years education including teachers, students, researchers and policy makers. www.childrens-mathematics.net

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